This Research was Sponsored in part by
The National Aeronautics and Space Administration
Research Grant No. NsG-110-61

METHODS OF WAFFLE PLATE SYNTHESIS

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June 1963

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CR-50,572

ABSTRACT

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The goal of structural synthesis is to automate the design process by use of digital computers. In general, the structural synthesis problem is, given a set of load conditions and side conditions for a structure, find by systematic means, a structure which will support the loads, satisfy the side constraints, and maximize the merit by which the structure is to be evaluated.

The structure dealt with is an integrally stiffened waffle like plate. Six design parameters are employed. These are total depth, thickness of the sheet, stiffener spacing in two directions, and stiffener width in two directions.

Several attempted methods of synthesis are discussed. Data is presented for five design problems.

ACKNOWLEDGEMENT

The authors express their appreciation and gratitude to:

The National Aeronautics and Space Administration, who sponsored the research program from which this report evolved. (Research Grant No. NsG - 110-61)

The Case Computing Center, and in particular Dr. George Haynam, for assistance in the computational work.

The many people of the Engineering Design Center and in particular the Engineering Synthesis Group for their willingness to discuss the project and for their suggestions.

SYMBOLS

a	x dimension of plate
b	y dimension of plate
b_x	Stiffener spacing in x direction
b_y	Stiffener spacing in y direction
D_1	Bending stiffness x direction
D_2	Bending stiffness y direction
D_3	Torsional stiffness
E	Modulus of Elasticity
H	Total depth of plate
N_x	Intensity of resultant normal force - x direction
N_{xy}	Intensity of resultant shear force
N_y	Intensity of resultant normal force - y direction
t	Distance of travel
t_s	Sheet thickness
t_{w_x}	Width of stiffener - x direction
t_{w_y}	Width of stiffener - y direction
W	Total weight of plate
δ	Steep descent increment
ϵ	Constraint surface tolerance
μ	Poisson's Ratio
ρ	Weight density
σ_o	Yield stress
φ_p	Direction cosines

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Chapter I

INTRODUCTION

The goal of structural synthesis is to automate the design process by use of digital computers. In general the structural synthesis problem is, given a set of load conditions and side conditions for a structure, find by systematic means a structure which will support the loads, satisfy the side conditions and maximize the merit by which the structure is to be evaluated. The side conditions may or may not dictate the type of structure to be used. At this stage in the development of synthesis techniques, it is assumed that the type of structure is determined by the side conditions or that several separate studies may be made on different types of structures. In most cases some of the design parameters are also predetermined.

In the case presented here the type of structure which has been selected is a waffle-like plate with integral orthogonal stiffeners. The successful development of a synthesis capability to minimize the weight of symmetric waffle plates has been reported in Ref. 1. This study included three independent design parameters, stiffener spacing, sheet thickness and stiffener thickness. The work presented here extends this capability to a more general one with more design parameters. This extension allows better designs to be obtained and gives an indication of the problems involved in developing synthesis techniques for problems of higher order in the design parameters.

The number of design parameters employed here is six. These are stiffener spacing in two directions, width of both sets of stiffeners, the total depth of the plate and sheet thickness.

Chapter II

THE WAFFLE SYNTHESIS PROBLEM

The structure dealt with is the waffle plate shown in Fig. 1. The design parameters as shown here are sheet thickness (t_s), total depth (H), stiffener spacing in the x direction (b_x), stiffener spacing in the y direction (b_y), width of the x stiffener (t_{w_x}), and width of the y stiffener (t_{w_y}). The over-all dimensions of the plate a and b are fixed and so is the material of the plate. The plate is subjected to inplane loads only. The positive loads are tension in the x direction (N_x), tension in the y direction (N_y), and shear on the positive x face in the positive y direction (N_{xy}). These are shown in Fig. 2.

The structure may be required to carry several different combinations of loads. That is, the plate may be subjected to one set of loads N_x, N_y, and N_{xy} at one time and other sets of loads N_x, N_y, and N_{xy} at other times. An acceptably designed plate must carry all of these. The computer programs written will handle up to five load conditions.

The criterion of merit used for evaluation of the design is the minimization of the total weight. The total weight is given by

$$W = \rho ab H \left[1 - (1 - \frac{t_s}{H}) (1 - \frac{t_{w_x}}{b_y}) (1 - \frac{t_{w_y}}{b_x})\right] \qquad (2.1)$$

The analysis used to predict the behavior of the structure is given in Appendix I. In writing the computer programs this analysis has been put in the form of behavior functions(see Ref. 1). By rearranging the failure conditions for each mode of failure, behavior functions are obtained in which the condition for satisfactory behavior is that each behavior function must be less than or equal to one. In addition to satisfying the behavior function conditions the designs obtained must also satisfy the side conditions. These are also given in Appendix I.

To solve the problem given above it is convenient to think of it in terms of a design space. This design space is formed by using each design parameter as a coordinate in the space. When the problem is thought of in this form, the behavior functions and the weight function, become hypersurfaces in the space. The side constraints on the design parameters become hyperplanes. All of the above are of dimension n-1 where n is the number of design parameters (6). Since all of these are of dimension n-1 the term hyper- will be dropped and the terms surface and plane will be used here.

Pictured in this manner the design problem is a problem of moving in a design space along a path which will converge to the minimum weight design. In general, the procedure used in doing this is to start with a design which lies in the acceptable region of the space. This is a region where none of the constraints (behavior or side constraints) are violated by the design. Then move in a series of steps toward the optimum. Design space nomenclature is presented in Fig. 3.

All the redesign processes presented here are in the same form. These are methods of alternate step and steep descent. If the present design point is a free point some form of steep descent is used. This is a move which decreases the weight. A free point is one which is not in violation and not within a certain given tolerance of any of the constraints. The second type of move, the alternate step, is made from a bound point. This move changes the design while maintaining a constant weight. The object of this move is to obtain another free point. A bound point is one where the design lies within a given tolerance of one or more constraints.

All the moves in both steep descent and alternate step are made with the same means of modification. This is to add to each of the present design parameters a change which is calculated by multiplying the corresponding direction cosine by the distance of travel. The equation for this is

$$\left\{ D_p^{q+1} \right\} = \left\{ D_p^q \right\} + t \left\{ \varphi_p \right\} \qquad (2.2)$$

where the D_p^{q+1} are the new values of the design parameters; the D_p^q are present values of the design parameters; the φ_p are the direction cosines; and t is the distance of travel.

Chapter III

STEEP DESCENT

Three different directions of steep descent are used in the different programs. One is parallel to the t_s axis and is made by reducing the weight by an increment and solving for the corresponding value of t_s, holding the other design parameters constant. (See section 4.2) The second is along the negative to the gradient to the weight surface. (See Fig. 4) The components of the gradient are:

$$
\begin{aligned}
\frac{\partial W}{\partial t_s} &= \rho ab \left[\left(1 - \frac{t_{w_x}}{b_y}\right) \left(1 - \frac{t_{w_y}}{b_x}\right) \right] \\
\frac{\partial W}{\partial H} &= \rho\, ab \left[1 - \left(1 - \frac{t_{w_x}}{b_y}\right) \left(1 - \frac{t_{w_y}}{b_x}\right) \right] \\
\frac{\partial W}{\partial b_x} &= - \frac{\rho ab\, H}{b_x} \; \frac{t_{w_y}}{b_x} \left(1 - \frac{t_s}{H}\right) \left(1 - \frac{t_{w_x}}{b_y}\right) \\
\frac{\partial W}{\partial b_y} &= - \frac{\rho ab\, H}{b_y} \; \frac{t_{w_x}}{b_y} \left(1 - \frac{t_s}{H}\right) \left(1 - \frac{t_{w_y}}{b_x}\right) \qquad (3.1) \\
\frac{\partial W}{\partial t_{w_x}} &= \frac{\rho\, ab\, H}{b_y} \left(1 - \frac{t_s}{H}\right) \left(1 - \frac{t_{w_y}}{b_x}\right) \\
\frac{\partial W}{\partial t_{w_y}} &= \frac{\rho\, ab H}{b_x} \left(1 - \frac{t_s}{H}\right) \left(1 - \frac{t_{w_x}}{b_y}\right)
\end{aligned}
$$

The third is to move from the present design point toward the point in the space (satisfying side constraints only) where all design parameters are such as to give a minimum weight. This is the point of minimum sheet thickness, minimum total depth, minimum stiffener widths, and maximum stiffener spacing. This method is used only as a first steep descent direction in later programs, Alternate Base Planes and Tangent Plane Methods. (See Fig. 5).

The procedure for moving along all of the above directions is the same. First an arbitrary increment is picked (this is input data). Then the new design is calculated and checked for design parameter and behavior function violation. If no violation is found the design is accepted as the new design point. The increment is then doubled and the new trial design checked. If this is an acceptable design, it becomes the new design point. This process is continued until a violation is obtained. Here the distance of travel is cut in half the new trial design being half way between the present acceptable design and the last unacceptable design. This is continued until an acceptable design is obtained. The acceptable design becomes the new design point. Starting from this point the distance is cut in half again and this trial checked. If the design is good it is accepted and a new trial proposed again halfing the distance. If the design is not good a new trial is proposed by taking half the last distance to the last acceptable design. This, then, continues until either a new design point is obtained which is within a certain prescribed tolerance of a constraint or the distance of travel becomes too small to be significant. This last design is accepted as a bound point. (See Figs. 4 and 5)

It should be noted here that in using the last two methods of steep descent, it is possible to move away from several of the side constraints. For this reason when a design is bounded by any of these side constraints, it is not regarded as bound. These constraints are the upper bound on H, the lower bound on b_x and b_y and the compatibility bounds between t_{w_x} and b_y, and t_{w_y} and b_x.

Chapter IV

ALTERNATE STEP

4.1 Distances to Side Constraints

Since a new trial design is obtained from the equation

$$\left\{ D_p^{q+1} \right\} = \left\{ D_p^{q} \right\} + t \left\{ \varphi_p \right\} \tag{2.2}$$

the distance to any one of the design parameter bounds may be obtained by setting D_p^{q+1} equal to the bound value and solving for t using the present values of D_p^q and φ_p (The direction cosines φ_p where p goes from 1 to 6 correspond to t_s, H, b_x, b_y, t_{w_x}, and t_{w_y}, respectively). The distances to the bounds are given by the following equations:

Maximum Depth

$$t = \frac{H_{max} - H}{\varphi_2} \tag{4.1}$$

Maximum y Stiffener Spacing

$$t = \frac{(b_y)_{max} - b_y}{\varphi_4} \tag{4.2}$$

Minimum x Stiffener Spacing

$$t = \frac{b_x - (b_x)_{min}}{-\varphi_3} \tag{4.3}$$

or

$$t = \frac{b_x - t_{w_y}}{\varphi_6 - \varphi_3} \tag{4.4}$$

The first of these is to the fixed bound $(b_x)_{min}$ but the second is the compatibility bound between b_x and t_{w_y}. Both of these must be checked since either may give the shorter distance.

Similarly for the minimum y stiffener spacing

$$t = \frac{b_y - (b_y)_{min}}{-\varphi_4} \qquad (4.5)$$

or

$$t = \frac{b_y - t_{w_x}}{\varphi_5 - \varphi_4} \qquad (4.6)$$

Minimum Width of Stiffeners:

x direction

$$t = \frac{t_{w_x} - (t_{w_x})_{min}}{-\varphi_5} \qquad (4.7)$$

y direction

$$t = \frac{t_{w_y} - (t_{w_y})_{min}}{-\varphi_6} \qquad (4.8)$$

Minimum Sheet Thickness

$$t = \frac{t_s - (t_s)_{min}}{-\varphi_1} \qquad (4.9)$$

Minimum Depth

$$t = \frac{H - t_s}{\varphi_1 - \varphi_2} \quad \text{or} \qquad (4.10)$$

$$t = \frac{\frac{W}{\rho ab} - t_s}{\varphi_1} \quad \text{(when H is dependent variable)} \qquad (4.11)$$

$$t = \frac{H - \frac{W}{\rho ab}}{-\varphi_2} \quad \text{(when } t_s \text{ is the dependent variable)} \qquad (4.12)$$

The above distances do not all apply simultaneously. If one of the direction cosines which appears in the denominator corresponds to the dependent variable (i.e., the variable being used to adjust the weight so that it remains constant) the expression for t is not valid and is skipped. Since in the case of the H, t_s compatibility bound it is possible to encounter the bound when H or t_s is the dependent variable substitute distances are used here. When H is equal to t_s the plate is solid and

$$H = t_s = \frac{W}{\rho ab} .$$

It is also possible to encounter the stiffener spacing stiffener width compatibility bounds when one of these variables is the dependent variable, but this condition is encountered infrequently and is only taken care of by rejecting an incorrect design using a design parameter side constraint check.

The shortest of the applicable distances are the ones which bound the design parameters in the given direction. Two values are obtained; one in the positive and one in the negative direction. (See Fig. 6).

After the maximum distances of travel are obtained the trial distance is then selected and the new trial design computed. Two methods are used in the different programs to select this distance.

One is to use fixed fractions of the total distance of travel and the other is to use random fractions. Because of the fact that it is better to use a large number of random directions checking only a few designs along each direction, it is felt that using random fractions is the better method. This makes it possible to obtain a more complete coverage of the space.

4.2 Projection to Weight Surface

In all of the synthesis techniques described here but Compromise II, a projection is made from a proposed design point to the weight surface parallel to one of the coordinate axes. This axis is the axis of the dependent variable in alternate step. The value of this variable and then using the value of the proposed design of the other five variables in this equation.

These equations are the following:

$$t_s = \frac{\frac{W}{\rho\ ab} + H\left[-\frac{t_{w_x}}{b_y} - \frac{t_{w_y}}{b_x} + \left(\frac{t_{w_x}}{b_y} + \frac{t_{w_y}}{b_x}\right)\right]}{\left(1 - \frac{t_{w_x}}{b_y}\right)\left(1 - \frac{t_{w_y}}{b_x}\right)}$$

$$H = \frac{\frac{W}{\rho\ ab} - t_s\left(1 - \frac{t_{w_x}}{b_y}\right)\left(1 - \frac{t_{w_y}}{b_x}\right)}{1 - \left(1 - \frac{t_{w_x}}{b_y}\right)\left(1 - \frac{t_{w_y}}{b_x}\right)}$$

$$b_x = \frac{t_{w_y}\left(1 - \frac{t_{w_x}}{b_y}\right)}{\left[\left(\frac{W}{\rho\, ab} - t_s\right)/(H - t_s)\right] - \frac{t_{w_x}}{b_y}}$$

$$b_y = \frac{t_{w_x}\left(1 - \frac{t_{w_y}}{b_x}\right)}{\left(\frac{W}{\rho\, ab} - t_s\right)/(H - t_s) - \frac{t_{w_y}}{b_x}} \tag{4.13}$$

$$t_{w_x} = b_y \left\{ \frac{\left(\frac{W}{\rho\, ab} - t_s\right)/(H - t_s) - \frac{t_{w_y}}{b_x}}{\left(1 - \frac{t_{w_y}}{b_y}\right)} \right\}$$

$$t_{w_y} = b_x \left\{ \frac{\left(\frac{W}{\rho\, ab} - t_s\right)/(H - t_s) - \frac{t_{w_x}}{b_y}}{\left(1 - \frac{t_{w_x}}{b_y}\right)} \right\}$$

4.3 Direction Scaling

In the later programs, Alternate Base Planes Method and Tangent Plane Method, the random numbers obtained are not used directly to determine the unit random vector, but are first multiplied by a scale factor corresponding to the appropriate design parameter. (see Ref. 2) Two methods of scaling are used, but only the first is used in the tangent plane method.

The first method is to scale them according to the range on the design parameters. This does not take into account the compatibility bounds but only the fixed maximum and minimum bounds, thus the scale factors are:

$$H_{max} - (t_s)_{min} \quad \text{for } H \text{ and } t_s$$

$$(b_x)_{max} - (t_{w_y})_{min} \quad \text{for } b_x \text{ and } t_{w_y}$$

$$(b_y)_{max} - (t_{w_x})_{min} \quad \text{for } b_y \text{ and } t_{w_x}$$

The reason behind this type of scaling is to obtain a uniform distribution of search directions over a region which is bounded by differing dimensions. A two dimensional example is presented in Fig. 7. This figure does not present data but only illustrates the effect. With no scaling the directions are characterized by a uniform angular distribution about the design point. It is thus difficult to investigate designs near $(b_x)_{min}$ and $(b_x)_{max}$. The scaled directions yield more directions of travel in the longer dimension of the space and allow more designs to be investigated at the extremities of the parameters with the larger range.

The second method is to choose scale factors based on the relative magnitudes of the final design parameter of previous synthesis problems and on the range of the parameters of the present problem. These factors are the input to the program.

Chapter V

SYNTHESIS TECHNIQUES

5.1 Several different schemes were tried in an effort to obtain an efficient means of optimization. A discussion of these is presented here in order of their development. For the purposes of discussion these have been given names. These are Compromise II, Sheet Thickness Method, Modified Sheet Thickness Method, Alternate Base - Planes Method, Point Saving Method and Tangent Plane Method, all of these methods assume that relative minima with respect to the weight surface do exist and all employ some type of search technique for the alternate step move.

In each of the alternate step techniques a selection of a random direction in the space is used after this line is determined a selection of a distance along this line is made. Since the side constraints on the design parameters are all linear they are hyperplanes in the design space, the acceptable region with respect to the design parameter bounds does not possess relative minima. Because of this it is possible to solve for the maximum possible distance of travel by taking the minimum of the distances to the design parameter bounds. This is done separately in both the forward and reverse directions. Distances of travel for the redesign are then made as fractions of the distances determined as above.

5.2 Compromise II

This method is similar to the Compromise II method which is described in Ref. 1. The differences being the method of maximum distance selection, the accelerated steep descent, and the number of design parameters. A steep descent in the direction of the negative of the gradient to the constant weight surface at the present design point is used. (see Steep Descent) No distance of travel is solved for but an arbitrary increment is used. The alternate step is made by solving for chord distances across the constant weight surface in a plane passing through the t_s axis and a random vector in the base normal to the t_s axis.

This direction of steep descent is much better than the direction used in the sheet thickness method, i.e., parallel to the t_s axis. The alternate step is complicated and may yield indeterminate distances in cases where the chord is very close to lying in the weight surface. For these reasons no attempt was made to obtain data from this program and another means of alternate step was adopted.

5.3 Sheet Thickness Method

In this method the steep descent is made parallel to the t_s axis. The alternate step is made by changing the design parameters in the base normal to the t_s axis and then solving for the value of t_s to project the design on to the constant weight surface (see Section 4.2)

This method of steep descent was found to be very inefficient.

5.4 Modified - Sheet Thickness Method

This method combines the better features of Compromise II and the sheet thickness method. The gradient steep descent is used here with the alternate step projected from the base normal to the t_s axis.

This method is much better than the sheet thickness method, because of the steep descent. However, because of the method of projection, the alternate step does not give a uniform search of the weight surface. For this reason it is possible for the program to arrive at a design point where the search is primarily carried out over a small region of unacceptable designs. When this occurs it is practically impossible to find a new acceptable design point.

5.5 Alternate Base Planes Method.

In an attempt to solve the problem encountered in projecting from one base plane to the weight surface, the program was changed to alternately project from all six of the possible base planes. This program is the same as the previous one, with gradient steep descent and base plane alternate step except that the base plane used each time is changed. This is done in cyclic order starting with the base normal to the t_s axis and proceeding in the following order: t_s, H, b_x, b_y, t_{w_x} and t_{w_y} (see Fig. 8). This order is not used in the special case where $(H - \epsilon) \leq t_s \leq H$. In this case projection always takes place parallel to the t_s axis. The direction cosine of H is set to zero and consequently t_s will change by only a very small amount in alternate step. This is done in order to allow a search of only the b_x, b_y, t_{w_x}, and t_{w_y} parameters. The proper combination of these parameters must then be found so that the gradient steep descent will take the design away from the t_s, H compatibility bound.

The first steep descent which moves directly toward the minimum weight corner is used to avoid hitting the t_s, H compatibility bound and in most cases is successful. When it is not successful finding the right combination of b_x, b_y, t_{w_x}, and t_{w_y} can be time consuming.

This program is much more efficient than the ones described previously and is the one used to obtain the numerical results presented in the following section.

5.6 Tangent Plane Method

This method is another attempt to improve the search of the weight surface. Random directions in the design space (six components) are projected onto a plane tangent to the weight surface. The random vector is given by $\bar{R}$ and the tangent vector, $\bar{T}$, is then given by $\bar{T} = \bar{R} - (\bar{R} \cdot \bar{n})\,\bar{n}$, where n is the unit normal to the weight surface. Here again after the distance of travel is selected, the design must be projected in some manner from the tangent plane to the weight surface. The way in which it is done is the same as in the previous method. That is to project parallel to one of the coordinate axes, the axis used being changed each time.

5.7 Point Saving

Since all the synthesis methods used here are search techniques, it is desirable to make use of as much of the information obtained in this search as possible. In all of the methods described previously the only information which is used is the present design point and the location of the design parameter bounds.

In this method acceptable design points are stored and used to generate directions of travel. The directions are the vectors between the present design point and the stored design points. These directions are then used in the same manner as the random directions. This method is used in conjunction with both the tangent plane method and the alternate base planes method. This is done only after the search using random directions has failed in a sufficient number of directions, (10).

The motivation for the development of this method is the desire to be able to locate possible "pockets" or "troughs" in the acceptable region of the design space. These are regions where the designs continuously remain in the acceptable region below the occupied weight contour. The idea is that a direction of travel between two points in the same "pocket" or "trough" might serve to cause a search through the "pocket," or that a direction of travel from one pocket to the next would locate a pocket which had been previously searched but which might yield better designs, and would be difficult to locate by using random vectors.

This method caused no significant change in the operating efficiency.

Chapter VI

NUMERICAL RESULTS

Four synthesis problems are presented here. Results obtained by starting from two different design points are presented for each case. Each column in the applied loads matrix represents one load condition, i.e.

$$\begin{bmatrix} N \end{bmatrix} = \begin{bmatrix} N_{x_1} & N_{x_2} & \cdots & N_{x_n} \\ N_{y_1} & N_{y_2} & \cdots & N_{y_n} \\ N_{xy_1} & N_{xy_2} & \cdots & N_{xy_n} \end{bmatrix}$$

Similarly each column in the behavior function matrix represents the behavior for one load condition, i.e.

$$\begin{bmatrix} BF \end{bmatrix} = \begin{bmatrix} GY_1 \ (D_p) & GY_2 \ (D_p) & \cdots & GY_n \ (D_p) \\ SX_1 \ (D_p) & SX_2 \ (D_p) & \cdots & SX_n \ (D_p) \\ SY_1 \ (D_p) & SY_2 \ (D_p) & \cdots & SY_n \ (D_p) \\ GBF_1 \ (D_p) & GBF_2 \ (D_p) & \cdots & GBF_n \ (D_p) \\ LBP_1 \ (D_p) & LBP_2 \ (D_p) & \cdots & LBP_n \ (D_p) \\ LBX_1 \ (D_p) & LBX_2 \ (D_p) & \cdots & LBX_n \ (D_p) \\ LBY_1 \ (D_p) & LBY_2 \ (D_p) & \cdots & LBY_n \ (D_p) \end{bmatrix}$$

where

$GY\ (D_p)$ = gross yield

$SX\ (D_p)$ = stiffener yield (x direction)

$SY\ (D_p)$ = stiffener yield (y direction)

$GBF\ (D_p)$ = gross plate buckling

$LBP\ (D_p)$ = local sheet buckling

$LBX\ (D_p)$ = local stiffener buckling (x direction)

$LBY\ (D_p)$ = local stiffener buckling (y direction)

The dimensions of the plate are all given in inches and the weight in pounds. A behavior function is considered bound when it is within $1 \pm \epsilon$, and the design is acceptable when the behavior function is less than $1 + \epsilon$.

The upper bounds on the design parameter are

$$\left\{ D_p^{(U)} \right\} = \left\{ \begin{matrix} H \\ (H)_{max} \\ (b_x)_{max} \\ (b_y)_{max} \\ b_y \\ b_x \end{matrix} \right\}$$

The lower bounds on the design parameters are

$$\left\{D_p^{(L)}\right\} = \left\{\begin{array}{c}(t_s)_{min} \\ t_s \\ b_{Lx} \\ b_{Ly} \\ (t_{w_x})_{min} \\ (t_{w_y})_{min}\end{array}\right\}$$

In the above equations, $(b_x)_{max}$ and $(b_y)_{max}$ are the maximum stiffener spacings consistant with equivalent plate analysis and are taken to be a/5 and b/5 respectively; b_{Lx} and b_{Ly} are the larger of $(b_x)_{min}$ and t_{w_y}, and $(b_y)_{min}$ and t_{w_x} respectively. Note that $(H)_{max}$, $(t_s)_{min}$, $(t_{w_x})_{min}$, $(t_{w_y})_{min}$, $(b_x)_{min}$ and $(b_y)_{min}$ may be assigned based on fabrication limitations.

CASE (1-3)

INPUT DATA

$\rho = 0.101$ lbs/in^3 $\qquad$ $\sigma_o = 72$ ksi

$\mu = 0.32$ $\qquad$ $a = 40$ in.

$\epsilon = 0.0001$ $\qquad$ $b = 30$ in.

$\delta_o = 0.01$ $\qquad$ $E = 10.5 \times 10^3$ ksi

$$[N] = \begin{bmatrix} -0.30 \\ -0.40 \\ +0.20 \end{bmatrix}$$

$(t_s)_{min} = 0.005$ $\qquad$ $H_{max} = 0.80$

$(t_{w_x})_{min} = 0.010$ $\qquad$ $(b_x)_{min} = 2.00$

$(t_{w_y})_{min} = 0.010$ $\qquad$ $(b_y)_{min} = 2.00$

TRIAL DESIGNS

Point A		Point B	
t_s	= 0.30	t_s	= 0.30
H	= 0.80	H	= 0.80
b_x	= 5.00	b_x	= 5.00
b_y	= 5.00	b_y	= 5.00
t_{w_x}	= 4.00	t_{w_x}	= 0.25
t_{w_y}	= 4.00	t_{w_y}	= 0.25

CASE (1-3)

FINAL OUTPUT

Point A		Point B	
t_s	= 0.0385	t_s	= 0.0355
H	= 0.8000	H	= 0.7828
b_x	= 2.0001	b_x	= 2.0000
b_y	= 2.8465	b_y	= 2.0002
t_{w_x}	= 0.0778	t_{w_x}	= 0.0260
t_{w_y}	= 0.0656	t_{w_y}	= 0.0909
W	= 10.1305	W	= 9.5456

$$\left[BF_A\right] = \begin{bmatrix} 0.1486 \\ 0.0702 \\ 0.0875 \\ 1.0000 \\ 0.9992 \\ 0.0870 \\ 0.1763 \end{bmatrix} \qquad \left[BF_B\right] = \begin{bmatrix} 0.1609 \\ 0.0921 \\ 0.0799 \\ 0.9999 \\ 0.9999 \\ 0.9823 \\ 0.0711 \end{bmatrix}$$

CASE (1 - T)

INPUT DATA

$\rho = 0.160$ lbs/in^3	$\sigma_o = 120$ ksi
$\mu = 0.29$	$a = 70$ in.
$\epsilon = 0.0001$	$b = 40$ in.
$\delta_o = 0.005$	$E = 16 \times 10^3$ ksi

$$[N] = \begin{bmatrix} -0.80 \\ 0.00 \\ 0.00 \end{bmatrix}$$

$(t_s)_{min} = 0.005$	$H_{max} = 2.50$
$(t_{w_x})_{min} = 0.010$	$(b_x)_{min} = 2.00$
$(t_{w_y})_{min} = 0.010$	$(b_y)_{min} = 2.00$

TRIAL DESIGNS

Point A	Point B
$t_s = 0.200$	$t_s = 0.40$
$H = 2.50$	$H = 2.50$
$b_x = 6.00$	$b_x = 2.00$
$b_y = 4.00$	$b_y = 2.00$
$t_{w_x} = 1.00$	$t_{w_x} = 0.50$
$t_{w_y} = 1.00$	$t_{w_y} = 0.50$

CASE (1 - T)

FINAL OUTPUT

Point A		Point B	
t_s	= 0.0308	t_s	= 0.0447
H	= 1.0103	H	= 1.1494
b_x	= 4.8420	b_x	= 2.0058
b_y	= 2.0002	b_y	= 3.2638
t_{w_x}	= 0.0471	t_{w_x}	= 0.0404
t_{w_y}	= 0.0337	t_{w_y}	= 0.0100
W	= 27.114	W	= 28.578

$$\left[BF_A\right] = \begin{bmatrix} 0.1237 \\ 0.1237 \\ 0.0000 \\ 0.9967 \\ 0.9999 \\ 0.9567 \\ 0.0000 \end{bmatrix} \qquad \left[BF_B\right] = \begin{bmatrix} 0.1143 \\ 0.1143 \\ 0.0000 \\ 0.9805 \\ 0.9950 \\ 0.9768 \\ 0.0000 \end{bmatrix}$$

CASE (1 - SL)

INPUT DATA

ρ = 0.101 lbs/in^3

μ = 0.32

ϵ = 0.0001

δ_o = 0.005

σ_o = 72 ksi

a = 100 in.

b = 20 in.

E = 10.5×10^3 ksi

$$[N] = \begin{bmatrix} -0.50 \\ -0.50 \\ +0.50 \end{bmatrix}$$

$(t_s)_{min}$ = 0.005

$(t_{w_x})_{min}$ = 0.010

$(t_{w_y})_{min}$ = 0.010

H_{max} = 2.50

$(b_x)_{min}$ = 2.00

$(b_y)_{min}$ = 2.00

TRIAL DESIGNS

Point A

t_s = 0.30

H = 1.00

b_x = 10.00

b_y = 2.50

t_{w_x} = 1.25

t_{w_y} = 5.00

Point B

t_s = 0.50

H = 1.50

b_x = 4.00

b_y = 3.00

t_{w_x} = 1.00

t_{w_y} = 1.00

CASE (1 - SL)

FINAL OUTPUT

Point A		Point B	
t_s	= 0.0522	t_s	= 0.0499
H	= 0.9289	H	= 0.7901
b_x	= 2.6440	b_x	= 2.0330
b_y	= 2.4510	b_y	= 2.9877
t_{w_x}	= 0.0343	t_{w_x}	= 0.0305
t_{w_y}	= 0.0337	t_{w_y}	= 0.0444
W	= 15.243	W	= 14.844

$$\left[BF_A\right] = \begin{bmatrix} 0.2549 \\ 0.1077 \\ 0.1096 \\ 0.8530 \\ 1.0000 \\ 0.9773 \\ 0.9992 \end{bmatrix} \qquad \left[BF_B\right] = \begin{bmatrix} 0.2664 \\ 0.1208 \\ 0.1051 \\ 0.9954 \\ 0.9999 \\ 0.9432 \\ 0.4487 \end{bmatrix}$$

CASE (3-2)

INPUT DATA

$\rho = 0.276$ lbs/in^3 $\qquad$ $\sigma_o = 150$ ksi

$\mu = 0.283$ $\qquad$ $a = 70$ in.

$\epsilon = 0.0001$ $\qquad$ $b = 50$ in.

$\delta_o = 0.01$ $\qquad$ $E = 30 \times 10^3$ ksi

$$[N] = \begin{bmatrix} -0.30 & +0.50 & -1.00 \\ -0.30 & -0.60 & 0.00 \\ 0.00 & -1.00 & -0.40 \end{bmatrix}$$

$(t_s)_{min} = 0.005$ $\qquad$ $H_{max} = 0.50$

$(t_{w_x})_{min} = 0.010$ $\qquad$ $(b_x)_{min} = 2.00$

$(t_{w_y})_{min} = 0.010$ $\qquad$ $(b_y)_{min} = 2.00$

TRIAL DESIGNS

Point A		Point B	
t_s	= 0.40	t_s	= 0.40
H	= 0.50	H	= 0.50
b_x	= 5.00	b_x	= 2.00
b_y	= 5.00	b_y	= 2.00
t_{w_x}	= 4.00	t_{w_x}	= 1.95
t_{w_y}	= 4.00	t_{w_y}	= 1.95

CASE (3-2)

FINAL OUTPUT

	Point A		Point B
t_s	= 0.0474	t_s	= 0.0474
H	= 0.5000	H	= 0.5000
b_x	= 5.2688	b_x	= 2.0077
b_y	= 3.8928	b_y	= 5.1094
t_{w_x}	= 0.6604	t_{w_x}	= 0.8678
t_{w_y}	= 0.9544	t_{w_y}	= 0.3634
W	= 185.75	W	= 185.76

$$\left[BF_A\right] = \begin{bmatrix} 0.0158 & 0.2484 & 0.1111 \\ 0.0161 & 0.0268 & 0.0537 \\ 0.0155 & 0.0309 & 0.0000 \\ 0.8426 & 0.6407 & 1.0000 \\ 0.2607 & 0.3687 & 0.3580 \\ 0.0001 & -0.0002 & 0.0003 \\ 0.0000 & 0.0001 & 0.0000 \end{bmatrix}$$

$$\left[BF_B\right] = \begin{bmatrix} 0.0158 & 0.2485 & 0.1111 \\ 0.0160 & 0.0268 & 0.0536 \\ 0.0154 & 0.0309 & 0.0000 \\ 0.8430 & 0.6412 & 1.0000 \\ 0.0935 & 0.0480 & 0.2755 \\ 0.0000 & -0.0001 & 0.0002 \\ 0.0003 & 0.0006 & 0.0000 \end{bmatrix}$$

CASE (3-2)'

INPUT DATA

$\rho = 0.276\ \text{lbs/in}^3$ $\qquad \sigma_o = 150$ ksi

$\mu = 0.283$ $\qquad a = 70$ in.

$\epsilon = 0.0001$ $\qquad b = 50$ in.

$\delta_o = 0.01$ $\qquad E = 30 \times 10^3$ ksi

$$[N] = \begin{bmatrix} -0.30 & +0.50 & -1.00 \\ -0.30 & -0.60 & 0.0 \\ 0.00 & -1.00 & -0.40 \end{bmatrix}$$

$(t_s)_{min} = 0.005$ $\qquad H_{max} = 2.50$

$(t_{w_x})_{min} = 0.01$ $\qquad (b_x)_{min} = 2.00$

$(t_{w_y})_{min} = 0.01$ $\qquad (b_y)_{min} = 2.00$

TRIAL DESIGNS

Point A		Point B	
t_s	= 0.40	t_s	= 0.40
H	= 2.50	H	= 2.50
b_x	= 5.00	b_x	= 2.00
b_y	= 5.00	b_y	= 2.00
t_{w_x}	= 4.00	t_{w_x}	= 1.95
t_{w_y}	= 4.00	t_{w_y}	= 1.95

CASE (3-2)'

FINAL OUTPUT

Point A		Point B	
t_s	= 0.0432	t_s	= 0.0475
H	= 0.9866	H	= 1.0128
b_x	= 2.0504	b_x	= 2.0451
b_y	= 4.6627	b_y	= 9.0909
t_{w_x}	= 0.0726	t_{w_x}	= 0.1318
t_{w_y}	= 0.0287	t_{w_y}	= 0.0291
W	= 68.499	W	= 72.459

$$\left[BF_A\right] = \begin{bmatrix} 0.0350 & 0.2894 & 0.1570 \\ 0.0345 & 0.0576 & 0.1151 \\ 0.0354 & 0.0709 & 0.0000 \\ 0.8262 & 0.6167 & 0.9783 \\ 0.3552 & 0.1971 & 1.0000 \\ 0.0508 & -0.0846 & 0.1692 \\ 0.4586 & 0.9172 & 0.0000 \end{bmatrix}$$

$$\left[BF_B\right] = \begin{bmatrix} 0.0326 & 0.2645 & 0.1457 \\ 0.0325 & 0.0542 & 0.1085 \\ 0.0327 & 0.0653 & 0.0000 \\ 0.7616 & 0.5438 & 0.8987 \\ 0.3125 & 0.1357 & 1.0000 \\ 0.0149 & -0.0248 & 0.0497 \\ 0.4597 & 0.9194 & 0.0000 \end{bmatrix}$$

Chapter VII

DISCUSSION OF NUMERICAL RESULTS

7.1 Comparison with Symmetric Synthesis Results

Two of the cases presented here offer a comparison with the symmetric synthesis results reported in Ref. 8. These are cases (1-3), and (3-2). In Case (1-3) a weight saving of 13 percent is obtained from 10.94 lbs to 9.54 lbs. and in case (3-2) only a 0.87 percent weight saving is obtained. It is felt that in cases where the loading and the overall plate dimensions are more unsymmetric a higher weight saving could be obtained. By looking at Case (1-T) it is seen that unsymmetric designs are obtained. This case is loaded in one direction only. If the plate is forced to be symmetric it might be expected that there would be a large weight penalty. It should also be noted here that the stiffening in both final designs is in the longitudinal direction and that for point B, t_{w_y} is at its lower bound. In Case (1-SL) with a symmetric load and highly unsymmetric over all dimensions the final design points are also unsymmetric but points A and B are unsymmetric in opposite directions.

7.2 Influence of the Depth Parameter

All the cases presented here show the influence of H on the design of waffle plates. The maximum depth is set reasonably high in all cases except (1-3) and (3-2). In the cases with the large maximum depth it is seen that the full depth is not utilized in the final design and that in these cases the final design is limited by gross buckling, local plate buckling and stiffener buckling in at least one direction.

In case (1-3) for point A the full depth is utilized but for point B it is not but stiffener buckling is an active constraint for both designs.

In Case (3-2) with a low depth limit the full available depth is utilized but stiffener buckling is not active and the only active behavior constraint is gross buckling in the third load condition.

Case (3-2)' is the same as case (3-2) except for the large H bound in Case (3-2)'. Two facts should be noted about these cases. One is that by increasing the depth from 0.5 inches to approximately 1.0 inches it is possible to decrease the weight from 185.75 lbs to 68.5 lbs. but without using the full depth available. The other is that in case (3-2) only the third load condition is active but in Case (3-2)' stiffener buckling in the second load condition is active along with local plate buckling and gross buckling in the third load condition.

7.3 Convergence

Case (3-2) is the only case presented here where the two synthesis paths can be considered to have converged to the same design. At first glance this does not seem to be true but on examination of the $\frac{t_w}{b}$ ratios for both points they are the same.

$$\frac{t_{w_x}}{b_y} = 1.70 \qquad \frac{t_{w_y}}{b_x} = 1.81$$

This is because the final design is gross buckling limited and only depends on the equivalent plate stiffness. In this case any design with these ratios for t_{w_x}/b_y and t_{w_y}/b_x and the same values of the other design parameters which does not cause violation of other constraints would be an optimum.

The reason that Case (3-2) converged while the others did not is that it is bounded only by one constraint while the final designs obtained for the other cases are bounded by more than one constraint. In reasonable running times the program is thus not capable of moving in the alternate step mode once the design is highly constrained.

Chapter VIII

CONCLUSIONS

8.1 Capability of Program

The development of a synthesis capability for six parameter waffle plate design can be considered partially successful. It cannot be considered completely successful because of the inability to achieve convergence to the same design in all cases. However, the designs obtained can all be considered efficient designs.

8.2 Relative Minima

While it is not shown that relative minima do exist in the design space, it has been assumed that they do. This seems only likely since the existence of relative minima was shown by plotting in the three dimensional problem. (see Ref. 1) The inability to converge can be interpreted as the inadequacy of the methods of alternate step to solve the relative minima problem. This is not surprising when one examines the way in which the relative minima problem is handled. This is to randomly search a subspace of dimension $n - 1 = 5$. Thus reducing the problem of searching a sixth dimensional design space to one of searching a finite number of fifth order spaces. Keeping this in mind, what has happened here might be expected. That is if the relative minima pockets are large or if there is a large acceptable region near the optimum these methods would be expected to work with some success. This is seen to be true in case (3-2) where there are many optimum designs. In the other cases presented, more than one constraint is active and achieving a redesign which is not in violation is much more difficult. Thus it is not practical to randomly search an $n-1$ subspace when n is as high as six.

8.3 Alternate Step Methods

In order to solve the relative minima problem using search methods of alternate step, better methods of selecting the alternate step direction must be developed. A useful approach to this problem is to look at it from the point of view of eliminating as many undesirable designs as possible rather than trying to develop a method which will select the one best direction. Several observations can be made concerning random directions which should be considered in developing methods of this type.

The first is that the distance traveled along any direction should not be so great as to take the design from the acceptable region with respect to design parameter bounds. This is done here. (see Chapter IV).

The second is the projection of the direction on to the merit surface. If the directions are all selected in one plane, the angle between this plane and the tangent to the merit at the point in question has the effect of distorting the distribution of the selected directions. This is the reason for using six planes of projection in the Alternate Base Planes Method and the reason that six random direction cosines have to be used in the Tangent Plane method instead of only five.

The third is direction scaling. Since in any problem the design parameters may vary over different ranges in order to uniformly search the acceptable design parameter region the direction cosines must be scaled according to these variations. While the overall design parameter variation remains fixed, the acceptable design parameter variation may change during the solution of the problem because of their interdependence through the merit equation. This is the reason for adopting the second method of direction scaling mentioned in section 4.3. In this second method no definite criterion is adopted to select scale factors and therefore it relies on the intuition of the operator. It is thus dangerous in the sense that used with abandon the operator may be doing the design himself thus defeating the purpose of writing a synthesis program. This method does not have any added advantage and it is felt that a more definite criterion should be developed.

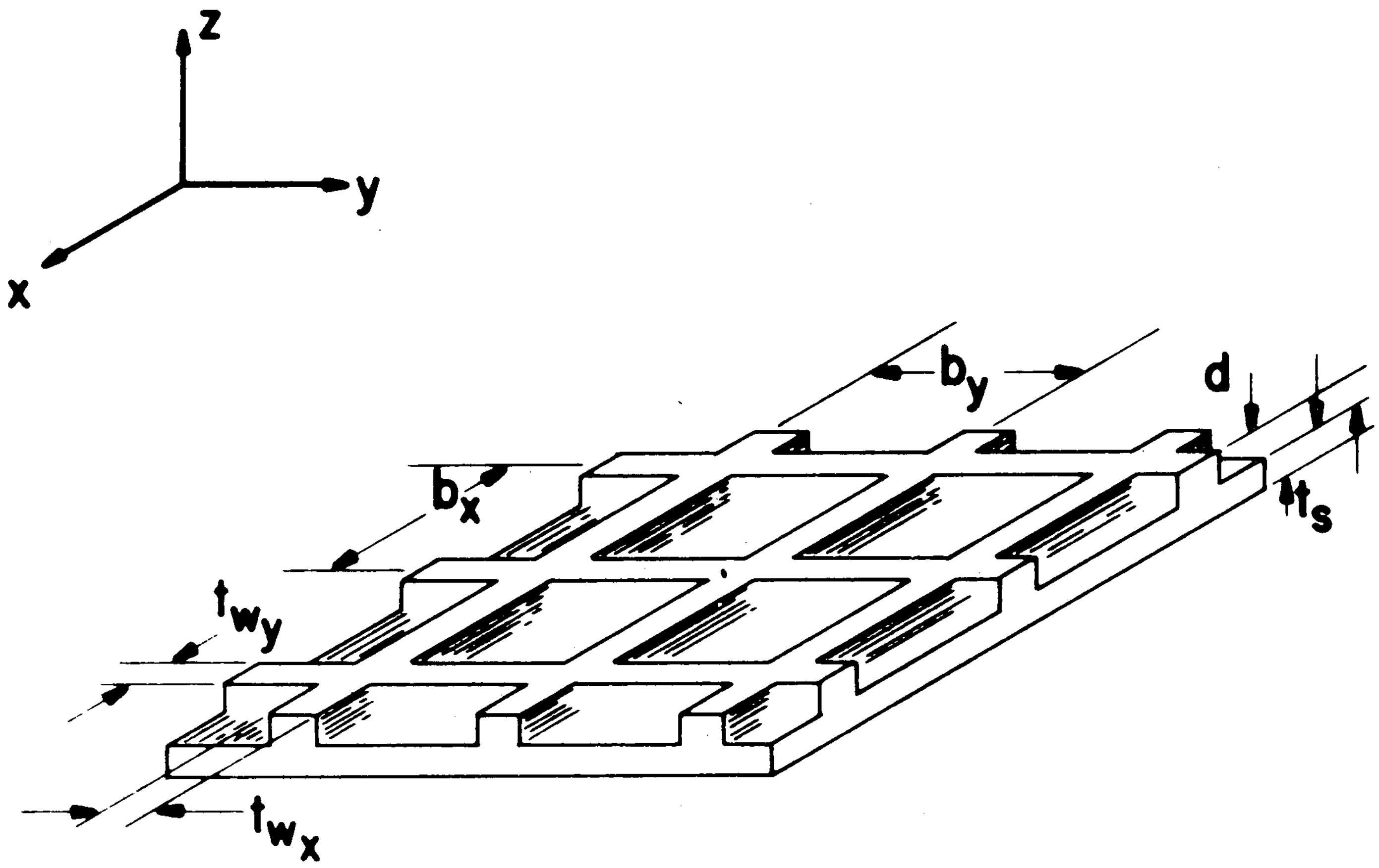
8.4 Efficiency of Operation

Aside from having the ability to solve the synthesis problem it is important to have an efficient method so that the amount of computer time consumed is not excessive. Running times for the results presented here ranged from four to eight hours per path on the Burroughs 220 Computer. This corresponds to from about one thousand to two thousand design cycles, which is excessive when compared with the three or four used in practice. As discussed above better method of alternate step would certainly improve this efficiency, but the methods of steep descent should also be investigated.

8.5 Steep Descent

In Ref. 1 it is pointed out that the convergence to a bound design in steep descent was slow. In the work done here an accelerated steep descent mode of travel is used. This is found to have no effect on the convergence time since the distances are in general short and the major portion of the running time is spent in refining the increment so that a bound design is obtained.

The gradient method of steep descent is the most efficient of the three used here. The corner direction is used only as a first try in order to avoid hitting the H, t_s compatibility bound. This bound is very difficult to move away from. The only way in which the program accomplished this was to arrive at a design, in alternate step, where the stiffener spacings were enough larger than the stiffener widths to allow gradient steep descent away from this bound.



INTEGRALLY STIFFENED WAFFLE PLATE FIG. 1

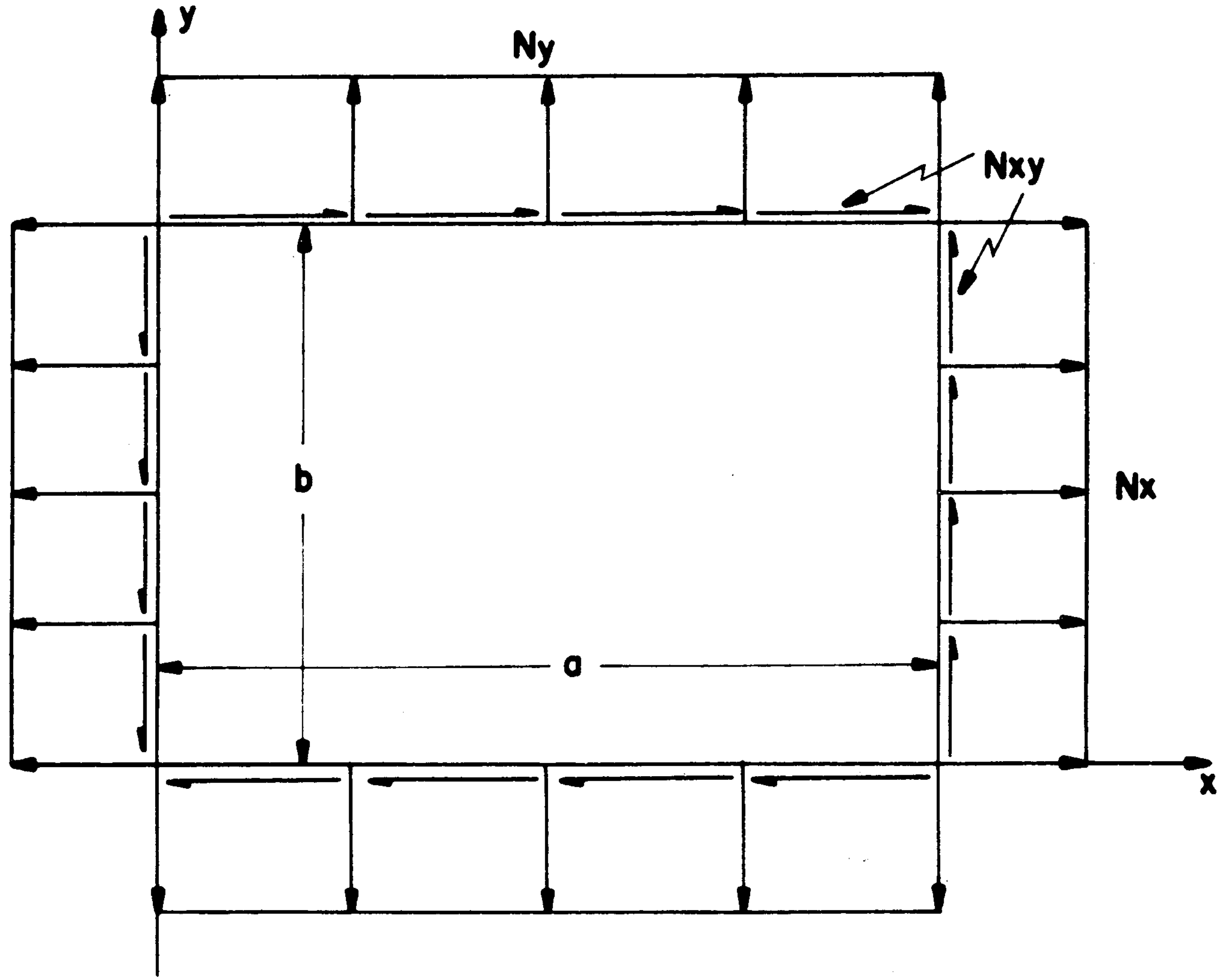


FIGURE 2 - APPLIED LOAD SIGN CONVENTION

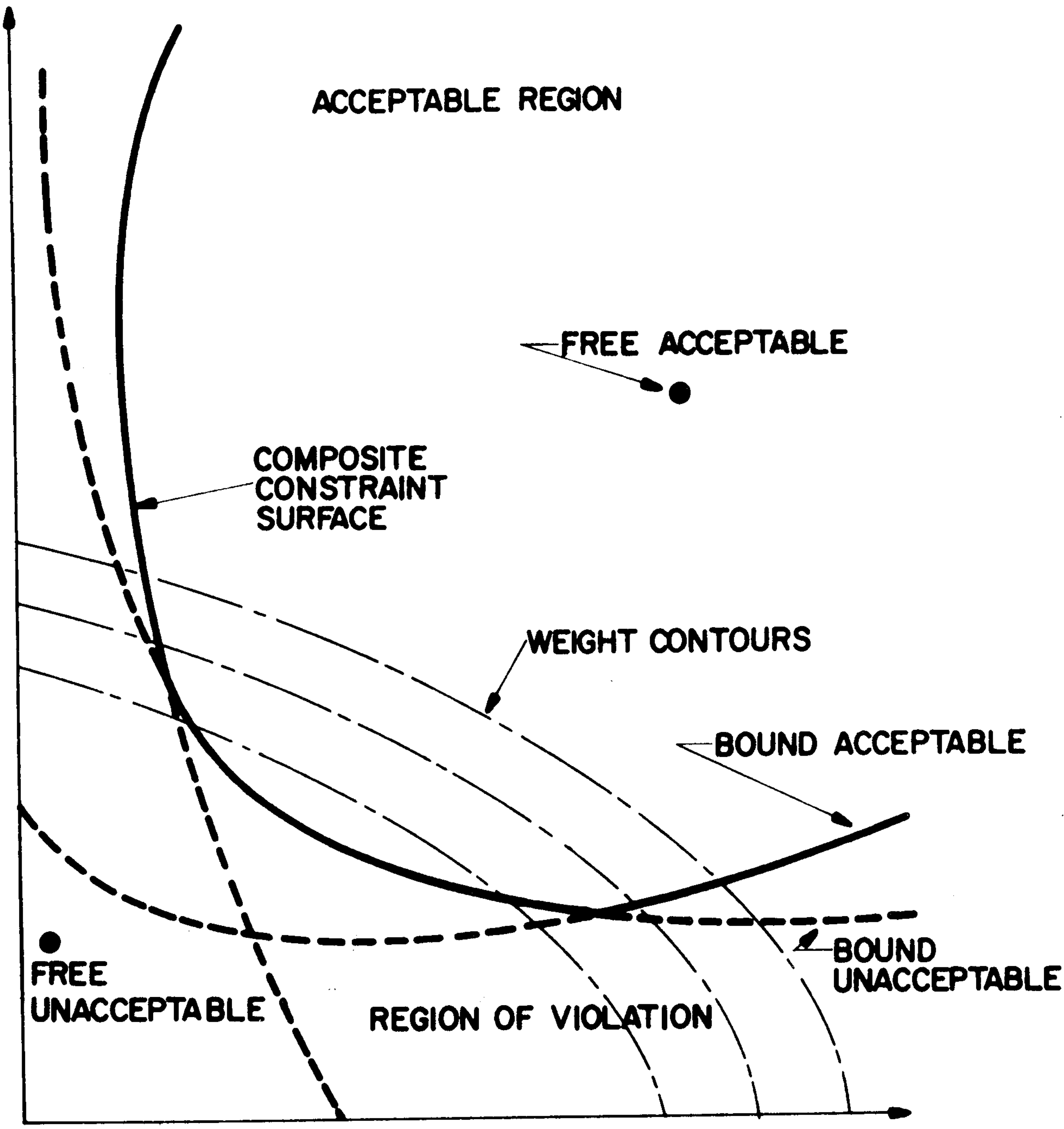


FIGURE 3 - DESIGN PARAMETER SPACE NOMENCLATURE

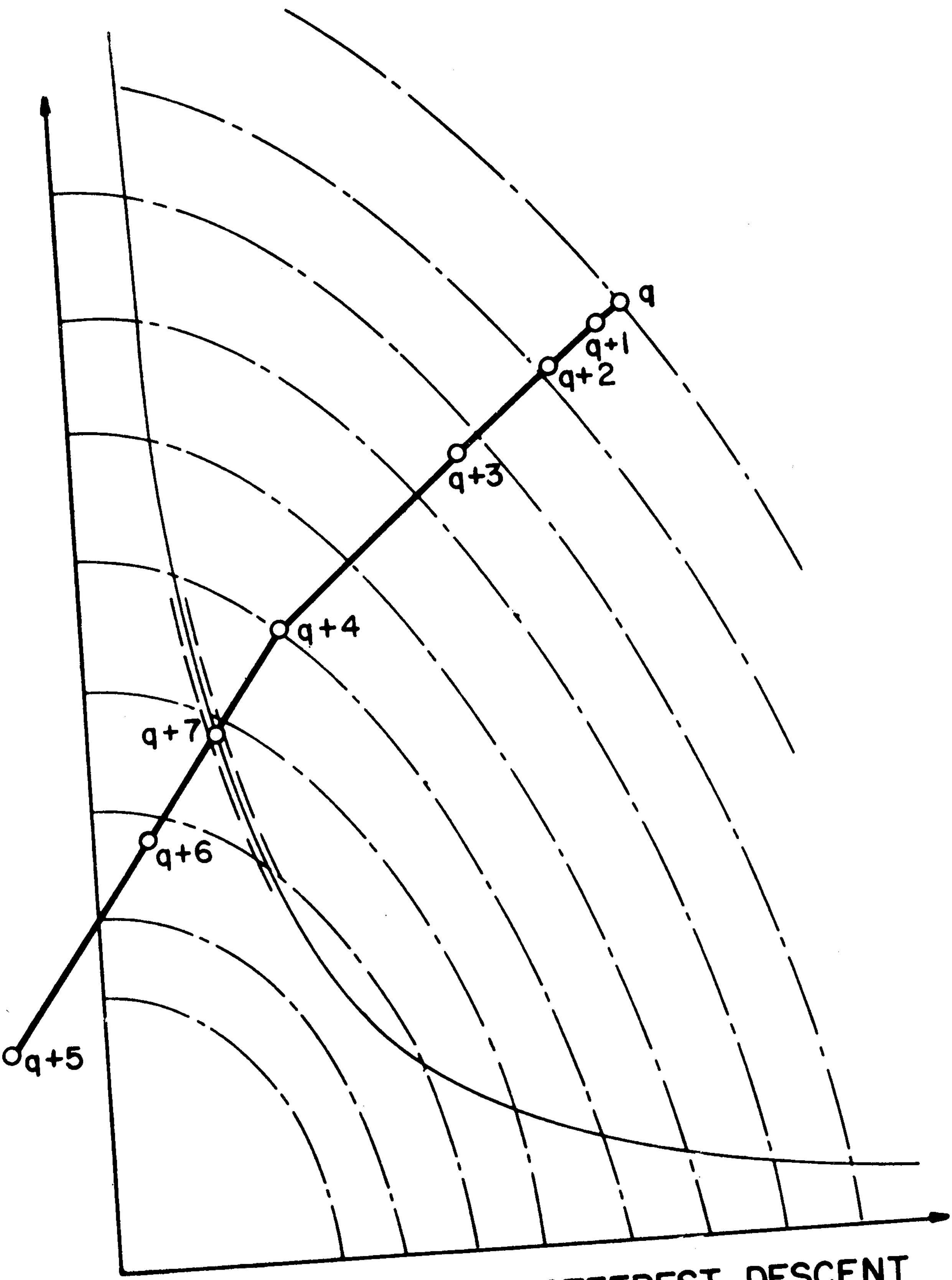


FIGURE 4 ACCELERATED STEEPEST DESCENT

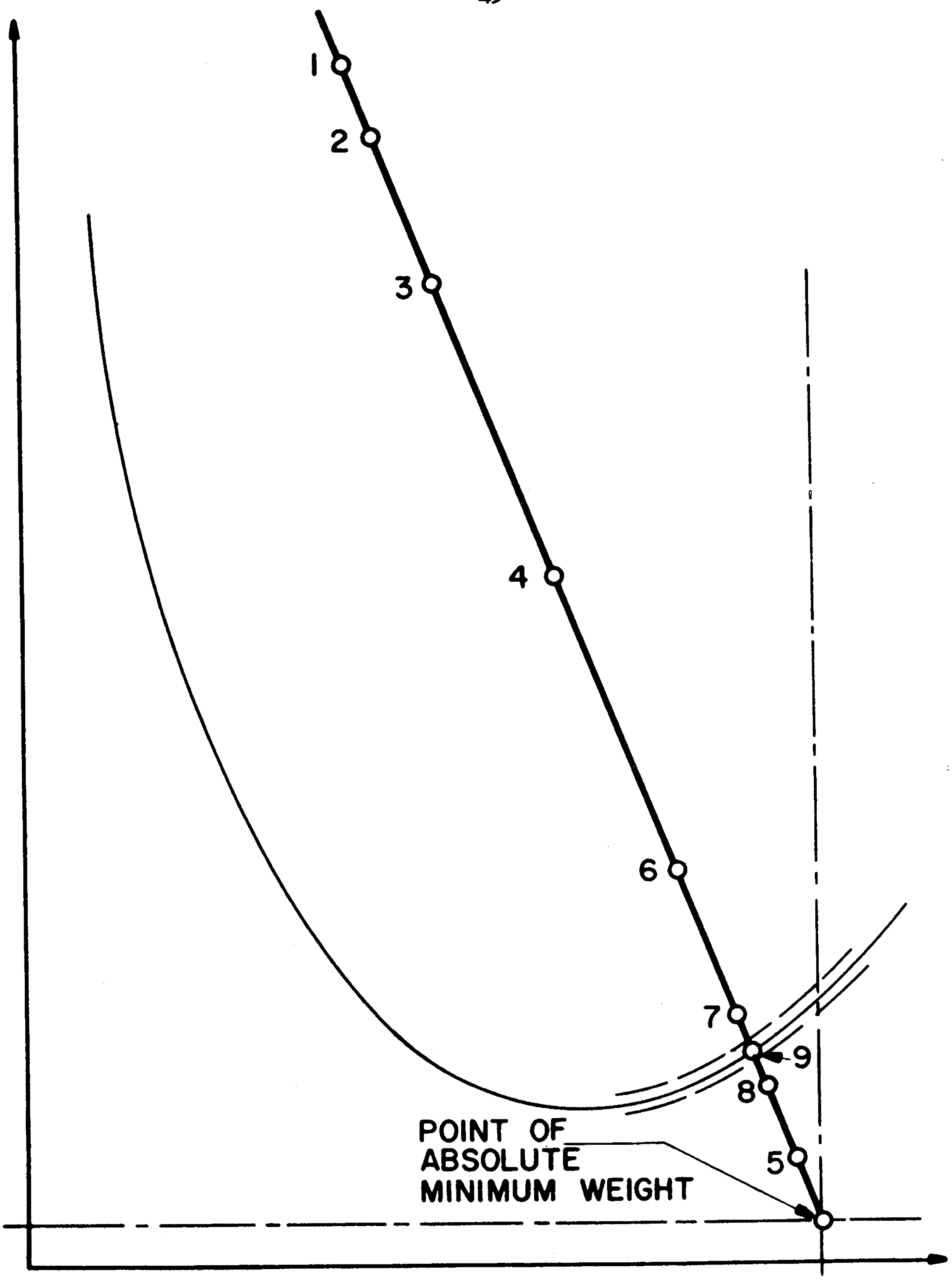


FIGURE 5 ACCELERATED INCREMENTATION— INITIAL STRAIGHT SHOT

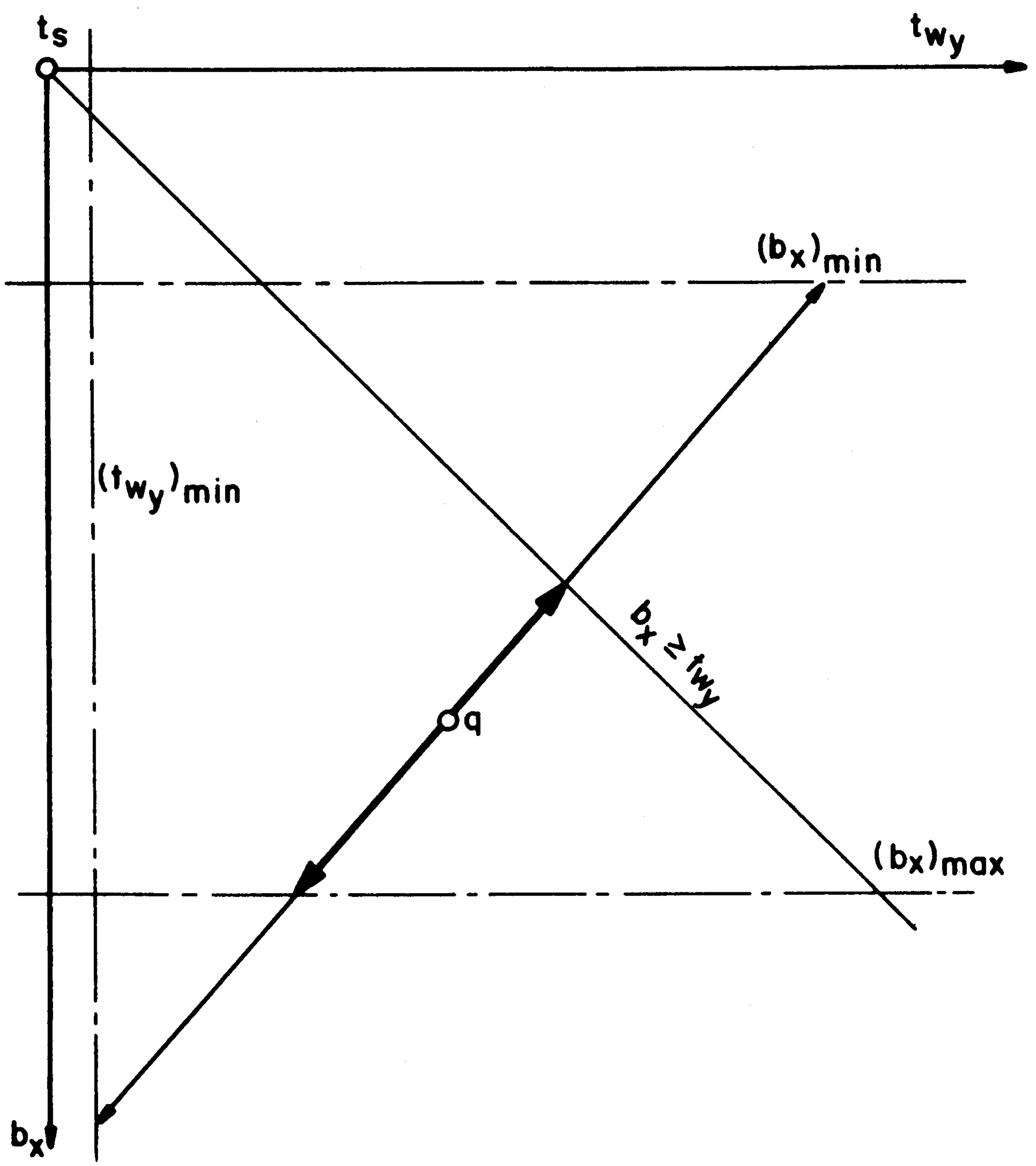
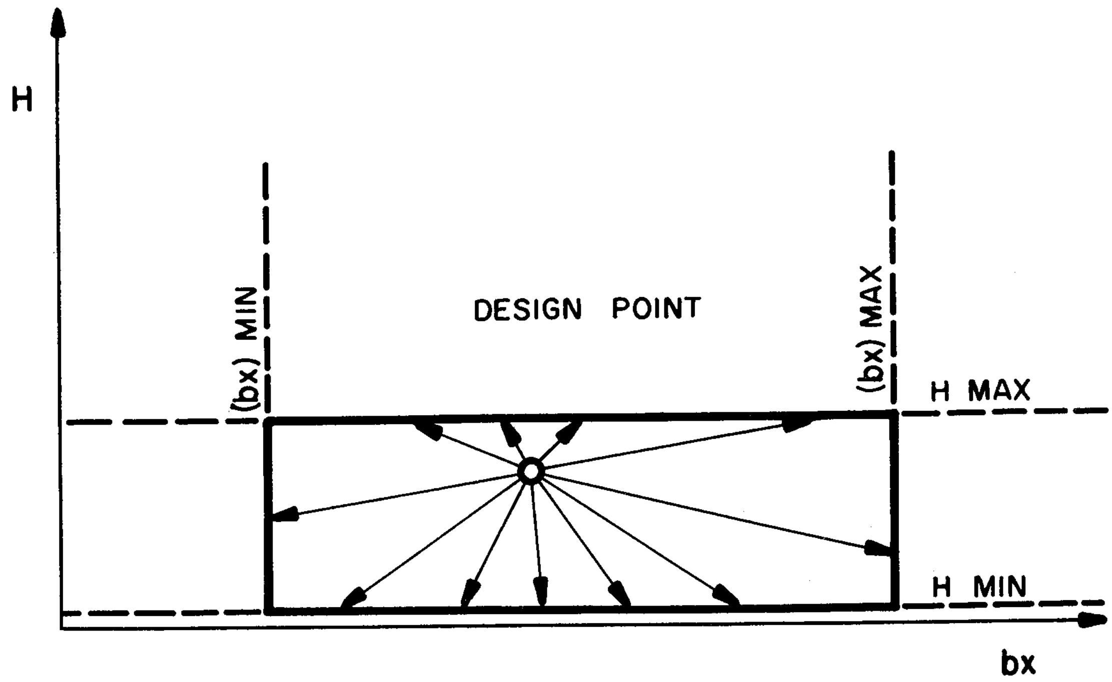
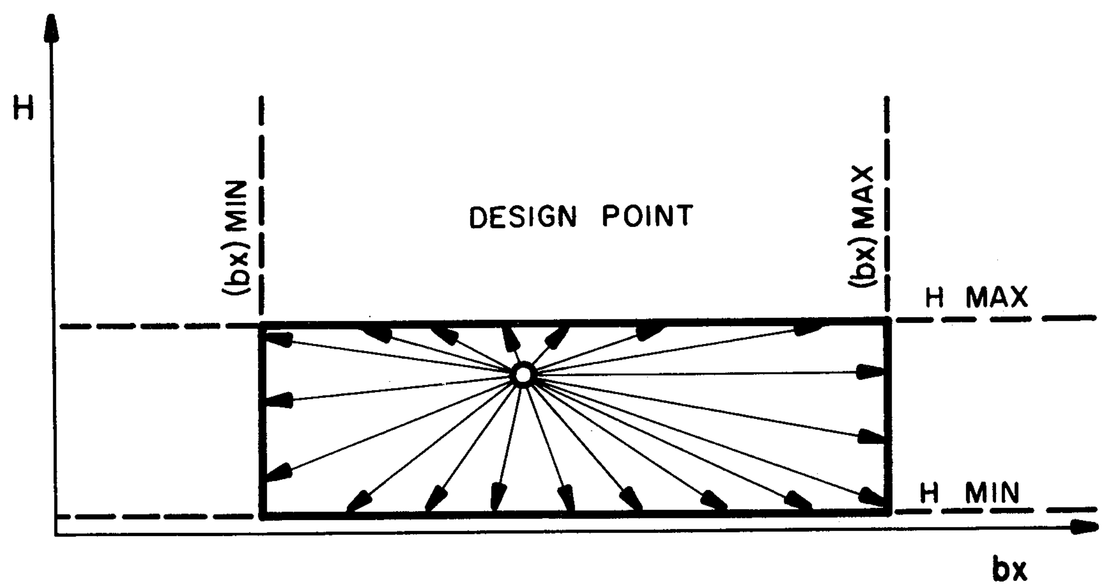


FIGURE 6 SIMPLIFIED ILLUSTRATION OF ALTERNATE BASE PLANES METHOD



RANDOM DIRECTIONS WITH NO SCALING



SCALED RANDOM DIRECTIONS

FIGURE 7 SEARCH DIRECTIONS

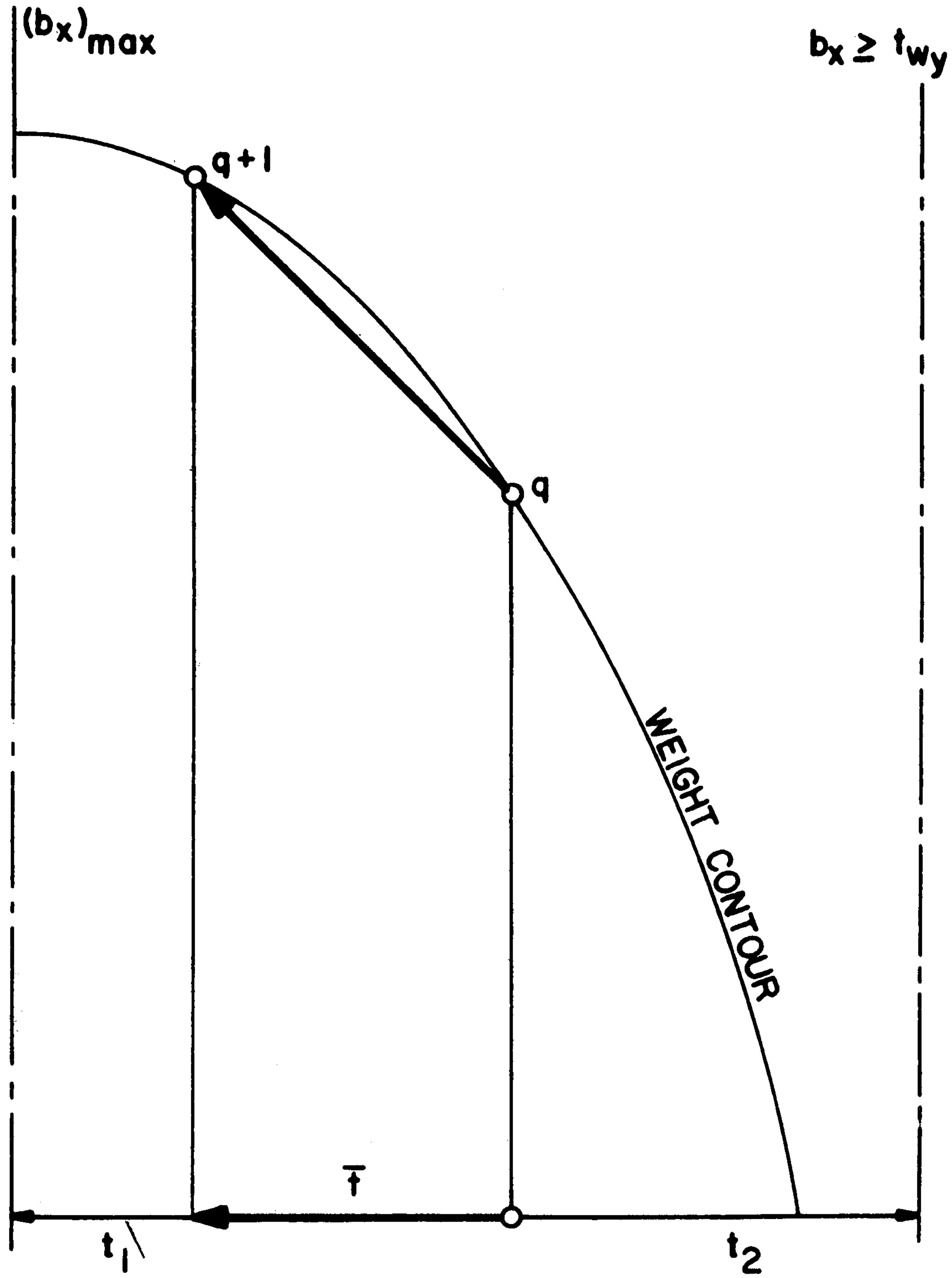


FIGURE 8 ALTERNATE STEP MODE OF TRAVEL

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Appendix I

PLATE ANALYSIS AND SIDE CONSTRAINTS

A.1.1 This appendix is a presentation of the equations necessary to write a computer program which analyzes a given rectangular waffle plate with orthogonal stiffeners which satisfies all side constraints. No detailed derivations or explanations of the equations are presented.

A.1.2 Orthotropic Plate Equations

By using equivalent elastic constants the gross instability of the waffle plate may be treated as that of an orthotropic plate with the same boundary conditions. Formulas for these constants have been derived in Ref. 3. The waffle plate has simply supported edges and is loaded by any combination of inplane loads N_x, N_y, and N_{xy}. The interaction expression proposed in Ref. 1. has been used for the work presented here. That is

$$\left(\frac{N_x}{(N_x)_{cr}} + \frac{N_y}{(N_y)_{cr}} + \frac{N_{xy}}{(N_{xy})_{cr}} \right)^2 = 1 \qquad (A.1)$$

For the case of biaxial loading with no shear the expression

$$\frac{N_x}{(N_x)_{cr}} + \frac{N_y}{(N_y)_{cr}} = 1 \qquad (A.2)$$

gives an exact solution to the orthotropic plate equation if both $(N_x)_{cr}$ and $(N_y)_{cr}$ are required to be in the same buckling mode.

To find $(N_x)_{cr}$ and $(N_y)_{cr}$ the above expression is written in the following forms:

$$N^x = \frac{(N_x)_r}{1 + \beta \left(\frac{a}{b}\right)^2 \left(\frac{n}{m}\right)^2} \tag{A.3}$$

$$N^y = \frac{(N_y)_r}{\frac{1}{\beta} + \left(\frac{a}{b}\right)^2 \left(\frac{n}{m}\right)^2} \tag{A.4}$$

where

$$(N_x)_r = \frac{\pi^2 \sqrt{D_1 D_2}}{b^2} \left[m^2 \left(\frac{b}{a}\right)^2 \sqrt{\frac{D_1}{D_2}} + \frac{2n^2 D_3}{\sqrt{D_1 D_2}} + \frac{n^4}{m^2} \left(\frac{a}{b}\right)^2 \sqrt{\frac{D_2}{D_1}} \right] \tag{A.5}$$

$$(N_y)_r = \frac{\pi^2 \sqrt{D_1 D_2}}{a^2} \left[n^2 \left(\frac{a}{b}\right)^2 \sqrt{\frac{D_2}{D_1}} + \frac{2m^2 D_3}{\sqrt{D_1 D_2}} + \frac{m^4}{n^2} \left(\frac{b}{a}\right)^2 \sqrt{\frac{D_1}{D_2}} \right] \tag{A.6}$$

$$\tilde{\beta} = \frac{N_y}{N_x} \tag{A.7}$$

and $(N_x)_{cr}$ and $(N_y)_{cr}$ are the values of the above for the critical mode given by m and n. (see Ref. 4) The critical values of m and n are obtained by finding their values such that the smallest positive value of N^x is obtained when N_x is compressive or N_y zero, or N^y when N_x is tensile or zero. D_1 and D_2 are the flexural rigidities and D_3 is the torsional rigidity.

Ref. 5 presents expressions for critical values of shear load N_{xy}:

$$N_{xy} = \frac{C_a \sqrt[4]{D_1 D_2^3}}{(\frac{b}{2})^2} \tag{A.8}$$

where $(C_a)_{cr}$ is the minimum of the two following expressions:

$$C_a = \frac{\pi^4}{128} \left\{ \frac{\left[\varphi\,(q+1,\,2)\right]^{1/2}}{2\,(q+1)\,\beta} \right\} \left\{ \left[\frac{q}{2q+1}\right]^2 \left[\frac{1}{9\,\varphi\,(q,\,1)} + \frac{9}{25\,\varphi\,(q,\,3)}\right] + \left[\frac{q+2}{2q+3}\right]^2 \left[\frac{1}{9\varphi\,(q+2,\,1)} + \frac{9}{25\varphi\,(q+2,\,3)}\right]\right\}^{-1/2} \tag{A.9}$$

which holds for symmetric buckling with q odd or antisymmetric with q even. For symmetric buckling with q even or anti-symmetric buckling with q odd.

$$C_a = \frac{\pi^4}{128} \left\{ \frac{1}{2\,(q+1)\,\beta} \right\} \left\{ \left[\frac{1}{9\,\varphi\,(q+1,\,1)} + \frac{9}{25\,\varphi\,(q+1,\,3)}\right] \left[\frac{q^2}{(2q+1)^2\,\varphi\,(q,\,2)} + \frac{(q+2)^2}{(2q+3)^2\,\varphi\,(q+2,\,2)}\right]\right\}^{-1/2} \tag{A.10}$$

where

$$\varphi\,(m,n) = (m\beta)^4 + \frac{2(m\beta)^2\,n^2}{\theta} + n^4$$

$$\beta = \frac{b}{a}\sqrt[4]{\frac{D_1}{D_2}}$$

$$\theta = \sqrt{\frac{D_1 \; D_2}{D_3^{\,2}}}$$

A.1.3 Equivalent Elastic Constants

The equivalent elastic constants used here are specialized from those given in Ref. 3. The assumptions and restrictions required to arrive at these expressions are stated in Ref. 1. The expressions for the flexural and torsional rigidities are:

$$D_1 = EH^3 \left[I_x - \frac{A_s^{\,2} \; A_x}{\bar{A}_s^{\,2}} \; (\bar{k}_x)^2 \right]$$

$$D_2 = EH^3 \left[I_y - \frac{A_s^{\,2} \; A_y}{\bar{A}_s^{\,2}} \; (\bar{k}_y)^2 \right]$$

$$D_3 = \frac{EH^3}{2} \left[2 \; \frac{\bar{I}_s^{\,2}}{\bar{A}_s^{\,2}} + I_{xy} \right]$$

where

$$I_x = \frac{1}{12\;(1 - \mu^2)} \left(\frac{t_s}{H}\right)^3 + \frac{I_{w_x}}{b_y \; H^3} + \frac{1}{4\; A_x^{\,2}\;(1 - \mu^2)} \; \frac{t_s}{H}$$

$$\left(\frac{A_{w_x}}{b_y \; H}\right)^2 + \frac{1}{4} \; \frac{A_{w_x}}{b_y \; H} \left[1 - \frac{1}{A_x} \left(\frac{A_{w_x}}{b_y \; H}\right)\right]^2$$

$$I_y = \frac{1}{12\;(1 - \mu^2)} \left(\frac{t_s}{H}\right)^3 + \frac{I_{w_y}}{b_x \; H^3} + \frac{1}{4\; A_y^{\,2}\;(1 - \mu^2)} \; \frac{t_s}{H}$$

$$\left(\frac{A_{w_y}}{b_x \; H}\right)^2 + \frac{1}{4} \left(\frac{A_{w_y}}{b_x \; H}\right) \left[1 - \frac{1}{A_y} \left(\frac{A_{w_y}}{b_x \; H}\right)\right]^2$$

$$\bar{A}_s^2 = A_x A_y - A_s^2$$

$$\bar{k}_x = \frac{1}{2 A_x} \frac{A_{w_x}}{b_y H}$$

$$\bar{k}_y = \frac{1}{2 A_y} \frac{A_{w_y}}{b_x H}$$

$$A_x = \frac{A_s}{\mu} + \frac{A_{w_x}}{b_y H}$$

$$A_y = \frac{A_s}{\mu} + \frac{A_{w_y}}{b_x H}$$

$$A_s = \frac{\mu}{1 - \mu^2} \frac{t_s}{H}$$

$$\bar{I}_s^2 = I_s \bar{A}_s^2 + A_s A_x A_y \bar{k}_x \bar{k}_y$$

$$I_{xy} = \frac{1}{6 (1 + \mu)} \left(\frac{t_s}{H}\right)^3$$

Then within these constants the following definitions are needed to obtain the equivalent elastic constants in terms of the design parameters:

$$\frac{I_{w_x}}{b_y H^3} = \frac{1}{12} \left(1 - \frac{t_s}{H}\right)^3 \frac{t_{w_x}}{b_y}$$

$$\frac{I_{w_y}}{b_x H^3} = \frac{1}{12} \left(1 - \frac{t_s}{H}\right)^3 \frac{t_{w_y}}{b_x}$$

$$\frac{A_{w_x}}{b_y H} = \left(1 - \frac{t_s}{H}\right) \frac{t_{w_x}}{b_y}$$

$$\frac{A_{w_y}}{b_x H} = \left(1 - \frac{t_s}{H}\right) \frac{t_{w_y}}{b_x}$$

$$I_s = \frac{\mu}{6\,(1 + \mu)} \left(\frac{t_s}{H}\right)^3$$

A.1.4 Local Buckling

Two modes of local buckling are considered. One is buckling of the backup sheet while the stiffener remains straight the other is buckling of the stiffener while the backup sheet remains flat.

Buckling of the backup sheet is considered as buckling of a rectangular isotropic plate with dimensions of the distances between the stiffeners and the thickness of the backup sheet. Here again the interaction expression (A1) is used. The critical values for the plate may be obtained by setting $D_1 = D_2 = D_3 = D$ and making proper substitutions for a and b in the expressions (A.3 - A.7) for the orthotropic plate. These become:

$$N_x = \frac{\pi^2 D}{(b_y - t_{w_x})^2} \left[m^2 \left(\frac{b_y - t_{w_x}}{b_x - t_{w_y}}\right)^2 + 2n^2 + \frac{n^4}{m^2}\left(\frac{b_x - t_{w_y}}{b_y - t_{w_x}}\right)^2 \right] F_x \tag{A.11}$$

$$N_y = \frac{\pi^2 D}{(b_x - t_{w_y})^2} \left[n^2 \left(\frac{b_x - t_{w_y}}{b_y - t_{w_x}}\right)^2 + 2m^2 + \frac{m^4}{n^2}\left(\frac{b_y - t_{w_x}}{b_x - t_{w_y}}\right)^2 \right] F_y \tag{A.12}$$

$$D = \frac{E\, t_s^3}{12\,(1 - \mu^2)}$$

where F_x and F_y are factors which take into account that only a portion of the load is carried by the sheet

$$F_y = 1 - (1 - \frac{H}{t_s}) \frac{t_{w_y}}{b_x}$$

$$F_x = 1 - (1 - \frac{H}{t_s}) \frac{t_{w_x}}{b_y}$$

$$\tilde{\beta}_s = \tilde{\beta} \frac{F_x}{F_y}$$

With similar substitutions the shear buckling equation becomes

$$N_{xy} = \frac{4 C_a D}{\tilde{b}^2} \qquad (A.13)$$

where in this case $\tilde{b}$ is the smaller of the values $b_x - t_{w_y}$ or $b_y - t_{w_x}$ and the C_a are the same as before with

$$\beta = \frac{\tilde{b}}{\tilde{a}} \qquad \theta = \tau$$

$$\rho\ (m,n) = \left[(m \frac{\tilde{b}}{\tilde{a}})^2 + n^2\right]^2$$

It should be noted here that all the shear is considered to be carried by the backup sheet and thus there is no reduction factor.

The stiffeners are considered as rectangular plates simply supported on three edges and free on the other. The critical values for buckling of the stiffener are given by the following expression: (see Ref. 6)

$$(N_x)_{cr} = \frac{-\pi^2 E\ (t_{w_x})}{12\ (1 - \mu^2)} \left[\frac{t_s}{t_{w_x}} + \frac{H - t_s}{b_y} \right] \frac{1}{(H - t_s)^2}$$

$$\left[\left(\frac{H - t_s}{b_y - t_{w_x}}\right)^2 + 0.425 \right] \qquad (A.14)$$

$$(N_y)_{cr} = \frac{-\pi^2\ E}{12\ (1 - \mu^2)} (t_{w_y})^3 \left[\frac{t_s}{t_{w_y}} + \frac{H - t_s}{b_x} \right] \frac{1}{(H - t_s)^2}$$

$$\left[\left(\frac{H - t_s}{b_y - t_{w_x}}\right)^2 + 0.425 \right] \qquad (A.15)$$

A.1.5 <u>Material Yield</u>

The material yield criterion is employed as a cutoff to the elastic buckling analysis. Three types of yield are possible. The stiffener in the x direction may yield; the stiffener in the y direction may yield or the backup sheet may yield. The x stiffener yield condition is

$$\sigma_o = \left| N_x \right| \frac{b_y}{b_y\ t_s + t_{w_x}\ (H - t_s)} \qquad (A.16)$$

The y stiffener yield condition is

$$\sigma_o = \left| N_y \right| \frac{b_x}{b_x\ t_s + t_{w_y}\ (H - t_s)} \qquad (A.17)$$

where the absolute value signs are used to take into account tension or compression. By substituting the principle stresses into the distortion energy yield criterion the condition for yield of the backup sheet becomes: (see Ref. 1)

$$\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2 = \sigma_o^2$$

Then in terms of the load and the design parameter this becomes

$$\frac{N_x^2\ b_y^2}{(b_y\, t_s + t_{w_x}\,(H - t_s))^2} - \frac{N_x\, N_y\, b_y\, b_x}{(b_y\, t_s + t_{w_x}\,(H - t_s))\,(b_x\, t_s + t_{w_y}\,(H - t_s))}$$

$$+ \frac{N_y^2\ b_x^2}{(b_x\, t_s + t_{w_y}\,(H - t_s))^2} + 3\left(\frac{N_{xy}}{t_s}\right)^2 = \sigma_o^2 \qquad (A.18)$$

A.1.6 Side Constraints

The side constraints which must be imposed on the plate designs arise from four different situations. These are

1. Bounds imposed by the limits of the manufacturing processes,
2. Bounds imposed by the applicability of the analysis,
3. Compatibility bounds to exclude physically absurd designs,
4. Bounds imposed by the use of the plate.

The bounds imposed by the limits of the manufacturing process are lower bounds on stiffener thickness and spacing. The bounds imposed by the limits on the analysis are the minimum number of stiffener.

Compatibility bounds arise where the stiffener width must be less than the corresponding stiffener spacing and the thickness of the sheet must be less than the total depth of the sheet.

The bound imposed by use of the plate is the total depth of the plate.

Appendix II

COMPUTER PROGRAM

A.2.2 The description of the computer program used for production is presented here. This is the program using the alternate base planes method. This and the other programs were written in Runcible (see Ref. 8) compiler language.

The required input for the program is the following list in Runcible notation:

A.2.2 INPUT DATA

$$\left.\begin{array}{lcl} Y1 & = & t_s \\ Y2 & = & H \\ Y3 & = & b_x \\ Y4 & = & b_y \\ Y5 & = & t_{w_x} \\ Y6 & = & t_{w_y} \end{array}\right\} \text{DESIGN PARAMETERS}$$

$$\left.\begin{array}{lcl} Y7 & = & E \\ Y8 & = & \rho \\ Y9 & = & \mu \\ Y10 & = & \epsilon \\ Y11 & = & \delta_o \\ Y12 & = & \sigma_o \\ Y13 & = & a \\ Y14 & = & b \end{array}\right\} \text{PREDETERMINED CONSTANTS}$$

Y26 - Y30 = N_{x_1} N_{x_2} ...

Y31 - Y35 = N_{y_1} N_{y_2} ...

Y36 - Y40 = N_{xy_1} N_{xy_2} ...

Y41 = $(t_s)_{min}$

Y42 = H_{max}

Y45 = $(t_{w_x})_{min}$

Y46 = $(t_{w_y})_{min}$

C68 = $(b_x)_{min}$

C69 = $(b_y)_{min}$

C70 = t_s scale factor

C71 = H scale factor

C72 = b_x scale factor

C73 = b_y scale factor

C74 = t_{w_x} scale factor

C75 = t_{w_y} scale factor

I13 = NUMBER OF LOAD CONDITIONS

A typical set of input data is presented in Fig. 9.

A.2.3 OUTPUT DATA

The design parameters are the same as above (Y1 - Y6). The behavior functions are the following:

Y53 - Y57 = GY (D_p)

Y58 - Y62 = SX (D_p)

Y63 - Y67 = SY (D_p)

Y68 - Y72 = GBF (D_p)

Y73 - Y77 = LBP (D_p)

Y78 - Y82 = LBX (D_p)

Y83 - Y87 = LBY (D_p)

I15 - Base Plane Indicator

Y25 - Weight

A typical set of output is presented in Fig. 10.

A.2.4. The operation of the program is presented in Figs. 11 and 12. The list below provides additional information concerning this operation.

G , refers to acceptable design

NG , refers to unacceptable design

I , is a cycle counter (1 is the initial cycle)

N , no

Y , yes

UB , means unbounded

δ , steep descent increment

δ_s , smallest steep descent increment allowed, a fraction of δ, 1/10, 1/100, 1/1000 or 1/10,000. This is an option on control switches 1 thru 4 respectively.

t , is the distance of travel to the side constraints.

When running the program one of the control switches one thru four should be set to determine the increment tolerance in steep descent. The program is in two segments with a small data processing portion in the first segment. Program control switch 5 will cause segment one to be read when the program is operating in segment two. After reading the data at the beginning of operation the data is then printed. All designs which are tested for behavior function violation in the alternate step mode are printed (locations Y1 - Y6 as above). The behavior function for these are not printed. In steep descent only the last bound design is printed and the behavior functions are printed for this design. A listing of the computer program follows.

```
1  + 200017006 +5040000000 +5050000000 +5150000000 +5150000000 +5140000000 +5140000000
1  + 200077001 +5530000000
1  + 200087007 +5027600000 +5028300000 +4710000000 +4910000000 +5315000000 +5270000000 +5250000000
1  + 200267003 -5030000000 +5050000000 -5110000000
1  + 200317003 -5030000000 -5060000000 +5000000000
1  + 200367003 +5000000000 -5110000000 -5040000000
1  + 200417002 +4850000000 +5050000000
1  + 200457002 +4910000000 +4910000000
1  + 300687002 +5120000000 +5120000000
1  + 300707001 +4910000000
1  + 300717001 +5010000000
1  + 300727001 +5110000000
1  + 300737001 +5110000000
1  + 300747001 +5010000000
1  + 300757001 +5010000000
1  - 100137001 +0000000003
```

FIGURE 9 TYPICAL CASE OF INPUT DATA

000 13004 RECALL RUNCIBLE PROGRAM FEB 19 17 39

+1200017007	+4947452284	+5049998960	+5152687526	+5138927718	+5066039628	+5095440880	+5530000000
-1200087007	+5027600000	+5028300000	+4710000000	+4910000000	+5315000000	+5270000000	+5250000000
-1200267005	-5030000000	+5050000000	-5110000000	+0000000000	+0000000000	+0000000000	+0000000000
-1200317005	-5030000000	-5060000000	+5000000000	+0000000000	+0000000000	+0000000000	+0000000000
-1200367005	+5000000000	-5110000000	-5040000000	+0000000000	+0000000000	+0000000000	+0000000000
-1200417006	+4850000000	+5050000000	+5214000000	+5210000000	+4910000000	+4910000000	+0000000000
-1300687002	+5120000000	+5120000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
-1100134084	+0000000003	+0100250000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000

-1200017006	+4947452284	+5049998960	+5152687526	+5138927718	+5066039628	+5095440880	+0000000000
-1200537003	+4915786293	+5024843201	+5011115009	+0000000000	+0000000000	+0000000000	+0000000000
-1200587003	+4916099975	+4926833292	+4953666584	+0000000000	+0000000000	+0000000000	+0000000000
-1200637003	+4915452701	+4930905402	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
-1200687003	+5084262951	+5064074420	+5110000500	+0000000000	+0000000000	+0000000000	+0000000000
-1200737003	+5026069304	+5036866252	+5035799244	+0000000000	+0000000000	+0000000000	+0000000000
-1200787003	+4696969139	-4716161523	+4732323046	+0000000000	+0000000000	+0000000000	+0000000000
-1200837003	+4643699037	+4687398074	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
-1100154043	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
-1200254031	+5318575402	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
-1100244075	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000

FIGURE 10 TYPICAL CASE OF OUTPUT

```
=99          9UNCI 235                                                    3133          13004
1       00000005500000000100000000000900000000090
                                          6. D = W= 7 8
2       RNG          RANDOM NUMBER
6 1                 0000 80410180019
6 1                 0001 80000340020
6 1                 0002 80000400018
6 1                 0003 80000420018
6 1                 0004 90000100014
6 1                 0005 80000140013
6 1                 0006 90001400014
6 1                 0007 00001490010
6 1                 0008 00000480002
6 1                 0009 80000120012
6 1                 0010 80000220012
6 1                 0011 00000304899
6 1                 0012 05000000000
6 1                 0013 09677214091
6 1                 0014 06250739481
6 1                 0015 03849370523
6 1                 0016 00000000007
6 1                 0017 06843575629
6 1                 0018 00000000000
6 1                 0019 00000000004
6 1                 0020 00033444352
6 1                 0021 00000304313
2       RNG     END OF RANDOM NUMBER
2       PCS          PROGRAM CONTROL SWITCH TEST
6 4                 0000 00000490009 81110400002 80000380005 00001450000
6 4                 0004 00000304899 80000100007 00000304899 00000000001
2       PCS          END OF PCS TEST SUBROUTINE
1       000                                   READ                      F
1       000                                   I7=0                      F
1       000                                   I8=0                      F
1       000                                   I9=0                      F
1       000                                   I10=0                     F
1       000                                   I11=3                     F
1       000                                   I12=1000                  F
1       000                                   I14=0                     F
1       000                                   I15=0                     F
1       000                                   I16=0                     F
1       000                                   I17=0                     F
1       000                                   I18=1                     F
1       000                                   I19=I17                   F
1       000                                   I20=0                     F
1       000                                   I21=0                     F
1       000                                   I23=0                     F
1       000                                   I24=0                     F
1       000                                   87,I0,0,1,I13-1,          F
1       087                                   C(79+I0)=Y(31+I0)/Y(26+I0) F
1       000                                   PUNCH C78 THRU C82        F
1       000                                   Y43=Y13/5                 F
1       000                                   Y15=Y8XY13XY14            F
1       000                                   Y44=Y14/5                 F
1       000                                   C53=Y11                   F
1       000                                   C5=(3.1415927)P2          F
1       000                                   C9=1./(1.-(Y9P2))         F
1       057                                   PUNCH Y1 THRU Y14         F
1       058                                   PUNCH Y26 THRU Y30        F
```

```
1      059                                                PUNCH Y31 THRU Y35        F
1      080                                                PUNCH Y36 THRU Y40        F
1      082                                                PUNCH Y41 THRU Y46        F
1      083                                                PUNCH C68 THRU C69        F
1      084                                                PUNCH I13 PUNCH I25       F
1      000                                                READ SEGMENT 2            F
1      000                                                SEGMENT 2                 F

1      00000005500000001000000000009000000000090
                                            6. D = W= 7 8
2      RNG        RANDOM NUMBER
6 1              0000 80410180019
6 1              0001 80000340020
6 1              0002 80000400018
6 1              0003 80000420018
6 1              0004 90000100014
6 1              0005 80000140013
6 1              0006 90001400014
6 1              0007 00001490010
6 1              0008 00000480002
6 1              0009 80000120012
6 1              0010 80000220012
6 1              0011 00000304899
6 1              0012 05000000000
6 1              0013 09677214091
6 1              0014 06250739481
6 1              0015 03849370523
6 1              0016 00000000007
6 1              0017 06843575629
6 1              0018 00000000000
6 1              0019 00000000004
6 1              0020 00033444352
6 1              0021 00000304313
2      RNG     END OF RANDOM NUMBER
2      PCS            PROGRAM CONTROL SWITCH TEST
6 4              0000 00000490009 81110400002 80000380005 00001450000
6 4              0004 00000304899 80000100007 00000304899 00000000001
2      PCS            END OF PCS TEST SUBROUTINE
1      037                                                C7=Y5/Y4                  F
1      000                                                C8=Y6/Y3                  F
1      000                                                C4=Y1/Y2                  F
1      000                                                C6=1.0-C4                 F
1      000                                                GO TO 38 IF I7U0          F
1      000                                                Y47=Y25/Y15               F
1      000                                                Y48=1.0-C7                F
1      000                                                Y49=1.0-C8                F
1      000                                                Y50=Y2-Y1                 F
1      000                                                Y51=Y3-Y6                 F
1      000                                                Y52=Y4-Y5                 F
1      000                                                Y53=(Y47-Y1)/Y50          F
1      000                                                GO TO (45+I15)            F
1      045                                                Y1=(Y47+Y2X(-C7-C8+C7XC8))/
1      045                                                Y48XY49                   F
1      000                                                GO TO 38                  F
1      046                                                Y2=(Y47-Y1XY48XY49)/
1      046                                                1.0-Y48XY49               F
1      000                                                GO TO 38                  F
1      047                                                Y3=Y6XY48/(Y53-C7)        F
1      000                                                GO TO 38                  F
1      048                                                Y4=Y5XY49/(Y53-C8)        F
```

```
1    000    GO TO 38                          F
1    049    Y5=Y4X(Y53-C8)/Y49                F
1    000    GO TO 38                          F
1    050    Y6=Y3X(Y53-C7)/Y48                F
1    038    I5=0                              F
1    000    I6=0                              F
1    000    C7=Y5/Y4                          F
1    000    C8=Y6/Y3                          F
1    000    C4=Y1/Y2                          F
1    000    C6=1.0-C4                         F
1    000    I8=I8+1                           F
1    000    I6=1 IF Y41 V Y1                  F
1    000    39,I0,0,1,2,                      F
1    039    I6=1IFY(2+I0)VY(42+I0)            F
1    000    I6=1IFY45VY5                      F
1    000    I6=1IFY46VY6                      F
1    000    I6=1IFY1VY2                       F
1    000    I6=1IFY5VY4                       F
1    000    I6=1IFY6VY3                       F
1    000    I6=1 IF C68 V Y3                  F
1    000    I6=1 IF C69 V Y4                  F
1    000    GOTO24IFI6U1IFI7U0IFI8U1          F
1    000    GO TO 25 IF I6U1 IF  I7U1         F
1    000    GO TO 23 IF I6U1 IF I7U0          F
1    000    I5=1IFY10VY1-Y41                  F
1    000    40,I0,1,1,2,                      F
1    040    I5=1IF Y10VY(42+I0)-Y(2+I0)F
1    000    I5=1IFY10VY5-Y45                  F
1    000    I5=1IFY10VY6-Y46                  F
1    061    PUNCH Y1 THRU Y6 IF I7 U 1 F
1    000    READ SEGMENT 1 IF
1    000    QPCSF,5Q U 1                      F
1    000    GO TO 64 IF Y10 V C55-C54
1    000    IF Y10 V Y2 -Y1                   F
1    062    1,I0,0,1,1,                       F
1    000    C(10+I0)=C6XC(7+I0)               F
1    001    C(12+I0)=(C6P3)XC(7+I0)/12 F
1    000    C14=(C4P3)/(1+Y9)X12              F
1    000    C15=Y9XC9XC4                      F
1    000    C16=(C4P3)XY9XC9/12               F
1    000    C17=C10+C9XC4                     F
1    000    C18=C11+C9XC4                     F
1    000    C19=(C17XC18)-(C15P2)             F
1    000    C20=(C16/Y9)+C12+
1    000    (C9XC4X(C10P2)/4X(C17P2))+
1    000    C10X((1.0-(C10/C17))P2)/4  F
1    000    C21=(C16/Y9)+C13+
1    000    (C9XC4X(C11P2)/4X(C18P2))+
1    000    C11X((1.0-(C11/C18))P2)/4  F
1    000    2,I0,0,1,1,                       F
1    002    C(1+I0)=Y7X(Y2P3)X(C(20+I0)
1    002    -((C15P2)X(C(10+I0)P2)/4X
1    002    C19XC(17+I0)))                    F
1    000    C3=Y7X(Y2P3)X(C16+(C15X
1    000    C10XC11/4XC19)+C14)               F
1    000    PUNCH C1 THRU C21                 F
1    000    C86=1.0-1.0/C4                    F
1    000    C84=1.0-C7XC86                    F
1    000    C85=1.0-C8XC86                    F
1    000    C22=QRT2E,C1/C2Q                  F
```

```
1  000   C23=1/C22                      F
1  000   C25=2XC3/QRT2E,C1XC2Q          F
1  000   Y16=Y13/Y14                    F
1  000   C24=QRT2E,C1XC2Q/C3            F
1  000   C21=QRT2E,C22Q                 F
1  000   12,I0,0,1,1,                   F
1  000   C31=C21/Y16                    F
1  000   C31=1/C31IFC31V1.0             F
1  006   11,I1,0,1,1,                   F
1  000   I2=0                           F
1  007   C(26+I1+2XI0)=C30              F
1  000   I2=I2+1                        F
1  000   8,I3,0,1,2,                    F
1  000   8,I4,1,1,3,                    F
1  008   C(31+I4+3XI3)=(((I2+I3)XC31)
1  008   P4)+(2X(((I2+I3)XC31XI4)P2)/
1  008   C24)+I4P4                      F
1  000   GO TO 9 IF I1 U 1              F
1  000   C41=QRT2E,C36Q/2X(I2+1)XC31F
1  000   C42=(I2/((2XI2)+1.))P2         F
1  000   C43=(1/9XC32)+9/25XC34         F
1  000   C44=((I2+2)/((2.XI2)+3))P2 F
1  000   C45=(1/9XC38)+9/25XC40         F
1  000   C30=C41/
1  000   QRT2E,(C42XC43)+(C44XC45)Q F
1  000   GO TO 10                       F
1  009   C41=1/2X(I2+1)XC31             F
1  000   C42=(1/9XC35)+9/25XC37         F
1  000   C43=(I2P2)/
1  000   (((2XI2)+1)P2)XC33             F
1  000   C44=((I2+2)P2)/
1  000   (((2XI2)+3)P2)XC39             F
1  000   C30=C41/QRT2E,C42X
1  000   (C43+C44)Q                     F
1  010   C(26+I1+2XI0)=C30IF I2 U1  F
1  000   GO TO 7 IFC(26+I1+2XI0)WC30F
1  011   CAS                            F
1  000   C(26+2XI0)=C(27+2XI0)IF
1  000   C(26+2XI0)VC(27+2XI0)          F
1  000   C24=1                          F
1  000   C21=1                          F
1  000   Y16=(Y3-Y6)/Y4-Y5              F
1  012   LOCAL CAS                      F
1  000   C31=1/Y16                      F
1  000   I0=0                           F
1  000   I0=1IFC31V1.0                  F
1  000   C20=Y7X(Y1P3)XC9/12            F
1  000   Y20=C28XC20X(C5P2)/
1  000   32X((Y(4-I0)-Y(5+I0))P2)       F
1  000   C31=Y14X((C1/C2)P0.25)/Y13 F
1  000   I0=1                           F
1  000   I0=0 IF C31V 1.0               F
1  000   Y19=(C26X(C5P2)X
1  000   (C(2-I0)XC(1+I0)P3)P0.25)/
1  000   32XY(13+I0)P2                  F
1  073   18,I1,0,1,I13-1,               F
1  000   GO TO 67 IF Y(26+I1) U 0.0
1  000   IF Y(31+I1)U0.0                F
1  000   C22=QRT2E,C1/C2Q               F
1  000   C23=1/C22                      F
```

```
1    000    C25=2XC3/QRT2E,C1XC2Q          F
1    000    PUNCHC22 THRU C25              F
1    000    Y16=Y13/Y14                    F
1    000    15,I0,0,1,1,                   F
1    000    C87=C(79+I1)                   F
1    000    C87=C87XC84/C85
1    000    IF I0U1                        F
1    000    GO TO 70 IF Y(26+I1)W0.0       F
1    074    C29=1.0                        F
1    000    C30=C87                        F
1    000    I17=0                          F
1    000    GO TO 76                       F
1    070    GO TO 74 IF Y(31+I1)U0.0       F
1    071    C29=1.0/C87                    F
1    000    C29=0.0 IF Y(26+I1)U0.0        F
1    000    C30=1.0                        F
1    000    I17=1                          F
1    076    I27=0                          F
1    000    I3=0                           F
1    013    I27=I27+5                      F
1    000    I27=1 IF Y(26+I1) U 0.0        F
1    000    HALT 2 IF I27 U 20             F
1    004    I3=I3+1                        F
1    000    I2=0                           F
1    003    C27=C28                        F
1    063    I2=I2+1                        F
1    000    C28=((C22X(I2/Y16)P2)
1    000    +(C25X(I3P2))+((C23X(Y16/I2)
1    000    P2)XI3P4))/C29+C30X
1    000    ((I3XY16/I2)P2)                F
1    000    PUNCH C28                      F
1    000    PUNCH I0 THRU I3               F
1    000    GO TO 77 IF C28V0.0            F
1    000    GO TO 3 IF I17 U 0             F
1    000    C27=C28 IF I3U1                F
1    000    GO TO 68                       F
1    077    GO TO 3 IF 0.0 V C27           F
1    000    GO TO 3 IF I2 U1               F
1    000    GO TO 3 IF C27 V C28           F
1    068    Y(52+I3)=C27                   F
1    000    I(27+I3)=I2-1                  F
1    000    GO TO 4 IF I27 V I3            F
1    000    PUNCH Y53 THRU Y(53+I27)       F
1    000    PUNCH I27 THRU I(28+I27)       F
1    000    I3=0                           F
1    079    I3=I3+1                        F
1    000    C28=Y(52+I3)                   F
1    000    GO TO 79 IF 0.0 V Y(52+I3)     F
1    000    5,I4,I3,1,I27-1,               F
1    000    I3=I4+1 IF C28 V Y(53+I4)      F
1    005    C28=Y(53+I4) IFC28VY(53+I4)F
1    000    GO TO 13 I FI3UI27 IF I27V1F
1    000    I2=I(27+I3)                    F
1    000    PUNCH I0 THRU I3               F
1    000    14,I4,0,1,1,                   F
1    000    C26=C(22+I4)X(I(2+I4)/
1    000    Y16)P2                         F
1    000    C27=C25X(I(3-I4)P2)            F
1    000    C28=C(23-I4)X((I(3-I4)P2)
1    000    XY16/I(2+I4))P2                F
```

```
1   000   C29=Y(14-I4)P2                  F
1   000   C29=((Y(4-I4)-Y(5+I4))P2)/
1   000   C(84+I4) IF I0 U 1              F
1   000   C30=QRT2E,C1XC2Q                F
1   000   C30=C20 IF I0U1                 F
1   000   PUNCH C26 THRU C30              F
1   000   Y16=1.0/Y16                     F
1   014   Y(17+I4+4XI0)=-C5XC30X
1   014   (C26+C27+C28)/C29               F
1   000   C22=1.0                         F
1   000   C23=1.0                         F
1   000   C25=2.0                         F
1   015   Y16=(Y3-Y6)/Y4-Y5               F
1   000   PUNCH Y17 THRU Y22              F
1   067   18,I0,0,1,1,                    F
1   000   Y(68+I1+5XI0)=(Y(26+I1)/
1   000   Y(17+4XI0))+(Y(31+I1)
1   000   /Y(18+4XI0))+(Y(36+I1)/
1   000   Y(19+I0))P2                     F
1   000   PUNCH Y68 THRU Y77              F
1   000   GO TO 18 IF (1.0-Y10)V
1   000   Y(68+I1+5XI0)                   F
1   000   GO TO 24 IF
1   000   A(1.0-Y(68+I1+5XI0))VY10        F
1   000   I5=1                            F
1   018   BYPASS                          F
1   000   C19=Y4-Y5                       F
1   000   C18=Y3-Y6                       F
1   000   C17=Y2-Y1                       F
1   000   16,I0,0,1,1,                    F
1   000   C(15+I0)=Y7X(Y(5+I0)P3)X
1   000   C9/12                           F
1   016   Y(23+I0)=-C5XC(15+I0)X
1   016   ((Y1/Y(5+I0))+C17/Y(4-I0))X
1   016   (((C17/C(18+I0))P2)+0.425)/
1   016   C17P2                           F
1   000   C10=Y4/((Y4XY1)+Y5XC17)         F
1   000   C11=Y3/((Y3XY1)+Y6XC17)         F
1   000   C12=C10P2                       F
1   000   C13=C10XC11                     F
1   000   C14=C11P2                       F
1   000   17,I0,0,1,I13-1,                F
1   000   Y(53+I0)=(((Y(26+I0)P2)XC12)
1   000   -(Y(26+I0)XY(31+I0)XC13)+
1   000   ((Y(31+I0)P2)XC14)+
1   000   3X(Y(36+I0)/Y1)P2)              F
1   000   Y(53+I0)=
1   000   QRT2E,Y(53+I0)/(Y12P2)Q         F
1   000   Y(58+I0)=AY(26+I0)XC10/Y12      F
1   017   Y(63+I0)=AY(31+I0)XC11/Y12      F
1   000   20,I0,0,1,1,                    F
1   000   20,I1,0,1,I13-1,                F
1   000   Y(78+I1+5XI0)=Y(26+I1+5XI0)/
1   000   Y(23+I0)                        F
1   000   GO TO 19 IF (1.0-Y10)V
1   000   Y(78+I1+5XI0)                   F
1   000   GO TO 24 IF
1   000   A(1.0-Y(78+I1+5XI0))VY10        F
1   000   I5=1                            F
1   019   GO TO 20 IF (1.0-Y10)V
```

```
1  019    Y(58+I1+5XI0)                       F
1  000    GO TO 24 IF
1  000    A(1.0-Y(58+I1+5XI0))VY10            F
1  000    I5=1                                F
1  020    BYPASS                              F
1  000    21,I1,0,1,I13-1,                    F
1  000    GO TO 21 IF (1.0-Y10)
1  000    VY(53+I1)                           F
1  000    GO TO 24 IF
1  000    A(1.0-Y(53+I1))VY10                 F
1  000    I5=1                                F
1  021    BYPASS                              F
1  000    I7=0                                F
1  000    I26=0.0                             F
1  081    I26=1 IF QPCSF,6Q U 1               F
1  000    C77=Y5/Y4                           F
1  000    C78=Y6/Y3                           F
1  000    Y25=Y8XY13XY14XY2X
1  000    (1.0-C6X(1.0-C7)X(1.0-C8))          F
1  000    GO TO 64 IF I10 V 0                 F
1  000    GO TO 65 IF I8 V 1                  F
1  000    C62=Y1-Y41                          F
1  000    C63=Y2-Y41                          F
1  000    C64=Y3-Y43                          F
1  000    C65=Y4-Y44                          F
1  000    C66=Y5-Y45                          F
1  000    C67=Y6-Y46                          F
1  000    GO TO 78                            F
1  064    C44=1.0-C7                          F
1  000    C45=1.0-C8                          F
1  000    C62=C44XC45                         F
1  000    C63=1.0-C62                         F
1  000    C64=-C44XC6XC8XY2/Y3                F
1  000    C65=-C45XC6XC7XY2/Y4                F
1  000    C66=C45XC6XY2/Y4                    F
1  000    C67=C44XC6XY2/Y3                    F
1  078    C41=0.0                             F
1  000    53,I0,0,1,5,                        F
1  053    C41=C41+C(62+I0)P2                  F
1  000    C41=QRT2E,C41Q                      F
1  000    54,I0,0,1,5,                        F
1  054    C(62+I0)=C(62+I0)/C41               F
1  065    GO TO 34 IF I5U1                    F
1  000    I18=1 IF QPCSF,1Q U 1               F
1  000    I18=10 IF QPCSF,2Q U 1              F
1  000    I18=100 IF QPCSF,3Q U 1             F
1  000    I18=10000 IF QPCSF,4QU1             F
1  000    GO TO 34 IF Y11/I18 W C53           F
1  000    56,I0,0,1,5,                        F
1  056    C(54+I0)=Y(1+I0)                    F
1  000    GO TO 23 IF I16U1                   F
1  022    C53=2.0XC53                         F
1  069    55,I0,0,1,5,                        F
1  055    Y(1+I0)=C(54+I0)-
1  055    C53XC(62+I0)                        F
1  000    GO TO 38                            F
1  023    C53=C53/2.0                         F
1  000    I16=1                               F
1  000    GO TO 69                            F
1  024    HALT 0 IF 1 W I8                    F
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```
1   000   GO TO 34 IF 1 W I8                  F
1   000   GO TO 23 IF I7U0                    F
1   025   GO TO 32 IF I11VI9                  F
1   041   I9=0                                F
1   000   I14=I14+1                           F
1   000   GO TO 25 IF I14U1                   F
1   026   I14=0                               F
1   000   HALT 1 IF I10 U I12                 F
1   000   I10=I10+1                           F
1   052   I15=I15+1                           F
1   000   I15=0 IF I15 W 6                    F
1   000   27,I0,0,1,5,                        F
1   000   C(46+I0)=(2XQRNGE,1Q)-1.0           F
1   027   C(46+I0)=C(46+I0)XC(70+I0)          F
1   000   I21=1                               F
1   000   I24=I24+1                           F
1   000   C50=0.0 IF I26U1                    F
1   000   C51=0.0 IF I26U1                    F
1   000   C47=0.0 IF Y10 V C55-C54
1   000   IF C63 V C62                        F
1   000   I15=0.0 IF Y10 VC55-C54             F
1   000   C48=0.0 IF Y10 V C56-C59            F
1   000   I15=5 IF Y10 V C56-C59              F
1   000   C49=0.0 IF Y10 V C57 -C58           F
1   000   I15=4 IF Y10 V C57-C58              F
1   051   C(46+I15)=0.0                       F
1   000   C43=0.0                             F
1   000   28,I0,0,1,5,                        F
1   028   C43=C43+C(46+I0)P2                  F
1   000   C43=QRT2E,C43Q                      F
1   000   66,I0,0,1,5,                        F
1   066   C(46+I0)=C(46+I0)/C43               F
1   000   PUNCH C46 THRU C51                  F
1   000   PUNCH I15 THRU I22                  F
1   000   C32=Y42-C55                         F
1   000   C33=Y43-C56                         F
1   000   C34=Y44-C57                         F
1   000   C35=C58-Y45                         F
1   000   C36=C59-Y46                         F
1   000   C37=C55-Y25/Y15                     F
1   000   C38=C56-C59-Y46                     F
1   000   C39=C57-C58-Y45                     F
1   000   C40=C54-Y41                         F
1   000   C41=C55-C54                         F
1   000   C41=(Y25/Y15)-C54 IF I15 U1F
1   000   C42=C56-C68                         F
1   000   C43=C57-C69                         F
1   000   Y53=C47                             F
1   000   Y54=C48                             F
1   000   Y55=C49                             F
1   000   Y56=-C50                            F
1   000   Y57=-C51                            F
1   000   Y58=0.0                             F
1   000   Y58=-C47 IF I15 U 0                 F
1   000   Y59=C51-C48                         F
1   000   Y59=0.0 IF I15 U5                   F
1   000   Y59=0.0 IF I15 U 2                  F
1   000   Y60=C50-C49                         F
1   000   Y60=0.0 IF I15U4                    F
1   000   Y60=0.0 IF I15 U 3                  F
```

```
1    000    Y61=-C46                           F
1    000    Y62=C46-C47                        F
1    000    Y62=C46IF I15 U 1                  F
1    000    Y62=0.0 IF I15 U 0                 F
1    000    Y63=-C48                           F
1    000    Y64=-C49                           F
1    000    44,I0,0,1,11,                      F
1    044    Y(53+I0)=0.0IFY(53+I0)U-0.0F
1    000    42,I0,0,1,11,                      F
1    000    C45=C(32+I0)/Y(53+I0)              F
1    000    C60=C45 IF Y(53+I0)V0.0            F
1    042    C61=C45 IF 0.0 V Y(53+I0)          F
1    000    30,I0,0,1,11,                      F
1    000    C45=C(32+I0)/Y(53+I0)              F
1    000    PUNCH C45                          F
1    000    GO TO 29 IF 0.0 W Y(53+I0)         F
1    000    C60=C45 IF C60 V C45               F
1    029    GO TO 30 IF Y(53+I0) W0.0          F
1    000    C61=C45 IF C45 V C61               F
1    030    BYPASS                             F
1    032    C32=C(60+I14)XQRNGE,2Q             F
1    000    33,I0,0,1,5,                       F
1    033    Y(1+I0)=C(54+I0)+
1    033    C32XC(46+I0)                       F
1    000    Y5=C77XY4 IF I26U1                 F
1    000    Y6=C78XY3 IF I26U1                 F
1    000    I9=I9+1                            F
1    000    GO TO 41 IF Y10 V A C32            F
1    000    GO TO 37                           F
1    034    PUNCH Y1 THRU Y6                   F
1    000    35,I0,0,1,6,                       F
1    035    PUNCH Y(53+5XI0) THRU
1    035    Y(52+I13+5XI0)                     F
1    043    PUNCH I15                          F
1    031    PUNCH Y25                          F
1    075    PUNCH I24                          F
1    000    36,I0,0,1,5,                       F
1    036    C(54+I0)=Y(1+I0)                   F
1    000    C53=Y11                            F
1    000    I7=1                               F
1    000    I9=0                               F
1    000    I14=0                              F
1    000    I16=0                              F
1    000    I24=0                              F
1    000    GO TO 26                          FF
```

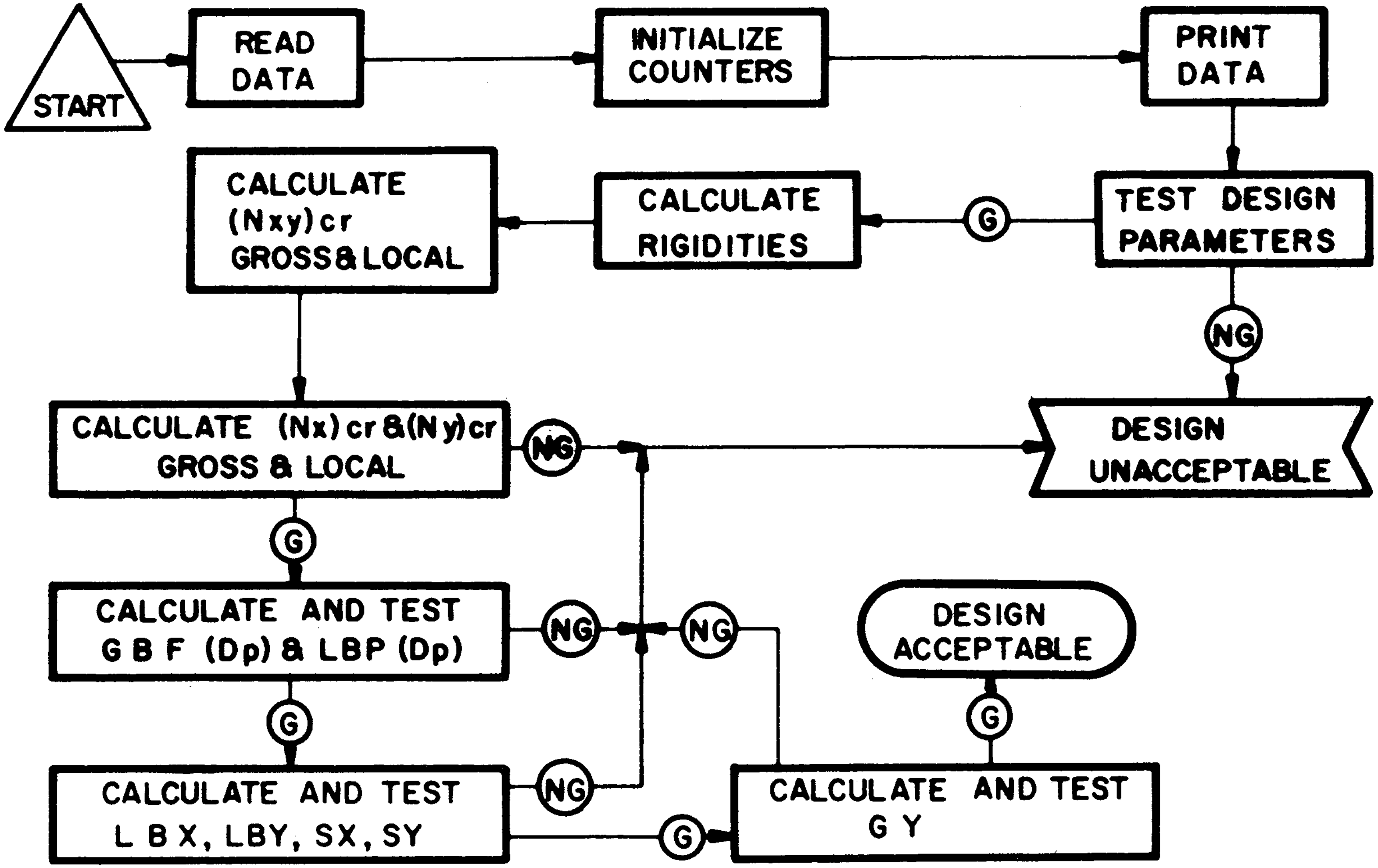

FIGURE 11 WAFFLE ANALYSIS

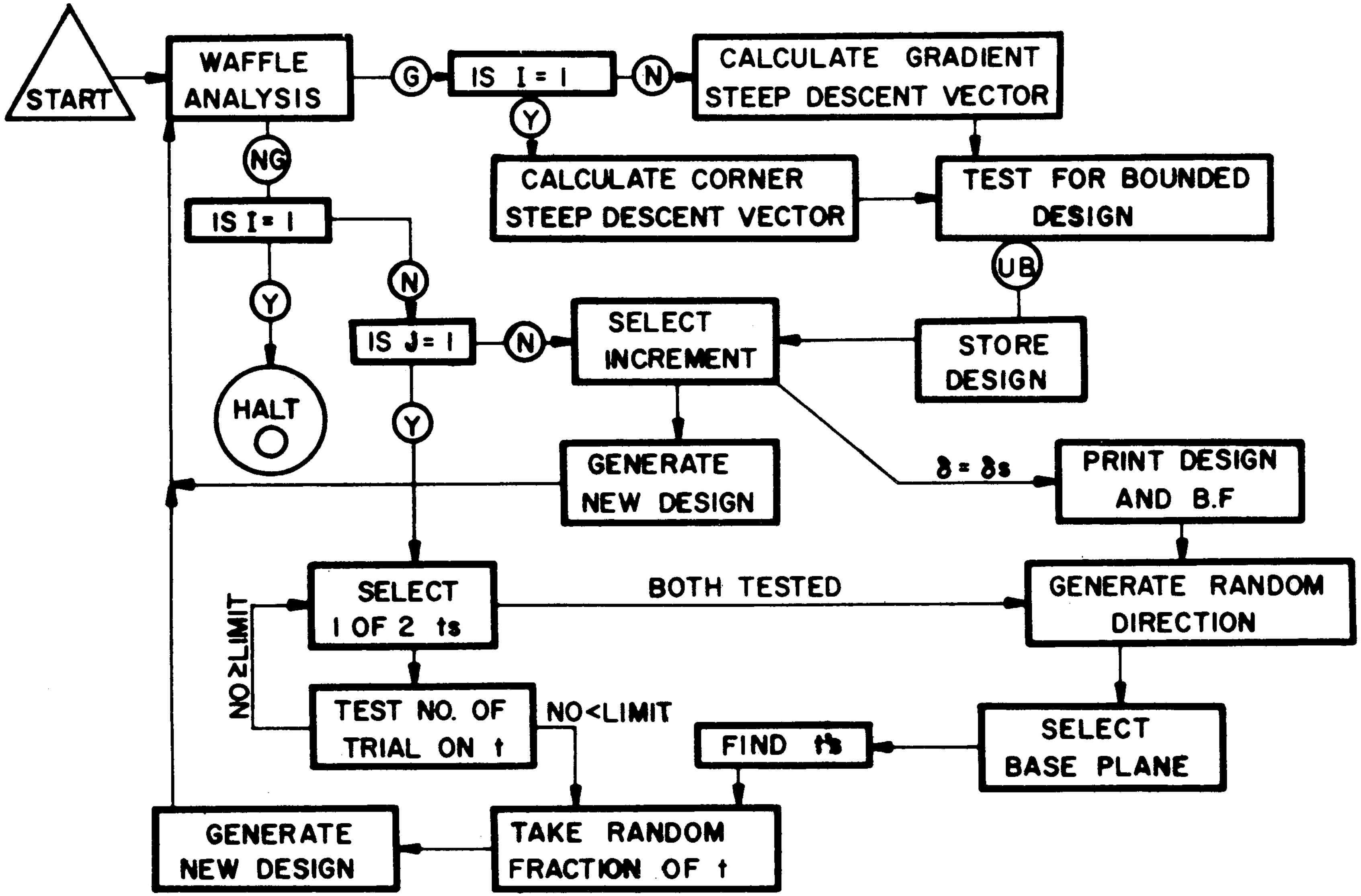

FIGURE 12 ALTERNATE BASE PLANES METHOD